Dynamics of R&D Competition: An Alternative setup

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Abstract

We build and analyze a stochastic model of R&D competition that combines a two-stage innovation race and disclosure decisions. We characterize a perfect Nash equilibrium in investment strategies and show that there exists particular conditions so that disclosure becomes profitable. In particular, either if the instantaneous profit of the first innovation is sufficiently small or sufficiently large, then both firms agree to sign a contract, before the begining of the R&D activity, commiting them to reveal their knowledge for free after the first period of competition.

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1 Introduction

Intellectual property (IP) and the patent system are important determinants of economic activity. For instance, the length of a patent may have important impacts on economic behaviors and activities. Effectively during the last twenty years the literature about R&D and patent races has become huge. Within this specific literature lots of papers appeared to sustain the actual innovation incentive scheme (the patent system or intellectual property), whereas lots of others did not and currently attack that system.

One complaint is IP rewards innovators beyond what is necessary to spur innovation. Another is that IP is a drag to innovation, rather than a spur, since it prevents inventions from being used efficiently, especially in creating further innovations. A third complaint is that some inventions should not be protected at all but, instead, be supported by public sponsors. However, the main critical aspect is the creation of a deadweight loss due to monopoly power. Thus alternative mechanisms for rewarding innovations have been created and analyzed by economists. The most famous alternative instrument is the prize system. By a *prize* we mean a payment funded out of general revenue that is made to a researcher conditional on delivering a specified invention. It has been shown (Kremer (1998), de Laat (1996)) that such a system can substantially improve social welfare by erasing the deadweight loss. But this scheme only seems credible either if the value of the invention is observable ex-post, or if the sponsor can verify that the researcher is investing exactly as he promised. Moreover, such a prize has to be funded through taxes supported by consumers, which corresponds to a distortion. Thus, if either cost or value of the innovation is truly observable to a sponsor, there may be a better mechanism than IP. Consistent with this view, Scotchmer (1999) justified patents by assuming that the cost and value are both unobservable. Finally, if the sole concern is to encourage innovation, then IP should last forever. On the contrary, if the sole concern is to avoid deadweight loss that occurs through monopoly prices, then IP should not exist at all. As regards these two rewarding systems, the basic problem appears clearly: on the one hand, IP creates a deadweight loss due to monopoly pricing and on the other hand, a prize creates distortions due to taxes.

Many authors have concentrated on other theoretical aspects such as multi-stage patent races or models including sequential innovations. The main motivation for that style of models is that *"innovations build upon each other and subsequent activity is directed toward improvements or applications of previous discoveries"* (Gallini and Scotchmer, 2002: p. 65). For instance Grossman and Shapiro (1987) built a two-stage (*research* and *development*) stochastic model which allows for technological progress. They found that competition is most intense when both firms are even and each has completed the initial phase of the research project. Horner (2004) and Aoki (1991) identify the existence of stationary Markov equilibrium strategies when two firms compete for an infinite number of innovations. Reinganum (1985) extended her former papers (1982, 1983) to a model with sequential innovations, in which one current incumbent firm and many challengers compete for the same innovation. The structure allows for the characterization of a perfect Nash equilibrium in investment strategies where the incumbent invests less than each challenger, such that the incumbency will tend to change hands frequently[1] (see also O'Donoghue (1998) for the analysis of a minimum innovation size required for patents). All these papers rely on memoryless technologies of research, whereas it seems intuitive that the later innovations could not be made without the earlier ones. In that sense, Doraszelski (2003) develops a model of an R&D race with knowledge accumulation. In contrast to the previous multi-stage race models where the follower slows down as he falls further behind, Doraszelski finds that a following firm, under certain conditions, may engage in catch-up behavior.

More recently, some authors have tried to analyze a specific behavior of investing firms in R&D under the so called patent system: disclosure of information as information sharing through license fee contracts[2]. Amongst them d'Aspremont et al. (2000) treat a model where two firms, that compete in the same R&D race, bargain over the disclosure of interim knowledge for an ultimate patentable invention. They find in equilibrium there is full disclosure from the informed firm to the uninformed firm. Regarding the sale of ideas, Anton and Yao (2002) show that partial disclosure of the idea to be sold might be used as a signal, allowing the seller to obtain payment based on the value of the remaining (undisclosed) know-how. In addition, Amir et al. (2003) make a comparison between cooperative and noncooperative behavior in a two-stage model of R&D. However they focus their research more on research joint venture (RJV) than on the effects of information sharing on investment strategies.

None of those articles looked at information sharing and sequential innovations jointly, as we do. We analyze a model that combines a two-stage innovation race with disclosure decisions. We characterize a perfect Nash equilibrium in investment strategies and we analytically prove the existence of particular conditions on the different parameters that give incentives to the firms to disclose fully their information rather than behave competitively. In order to

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[1] This refers to the notion of creative destruction introduced by Schumpeter in 1942, that is the incumbent enjoys temporary monopoly power but is soon overthrown by a more inventive challenger.

[2] Even in some contexts, such as computer sciences, we can observe free information revealing. For instance in the open source context where von Hippel (2005) estimates that more than 83000 software codes fell in the public domain in 2004.

keep things clear, we show graphically the existence of conditions under which the disclosure behavior involves higher expected profits than the competitive behavior. Especially, if the instantaneous profit of the first innovation is sufficiently small or sufficiently large, then both firms agree to sign a contract, before the beginning of the R&D activity, so that they engage to freely reveal their knowledge after the first period of competition.

Our paper is organized as follows: section 2 presents the basic model including a comparison between both behaviors (competitive and disclosure). Section 3 provides an alternative version of the model where the assumption of constant cost reduction amongst the two innovations is relaxed. We discuss the economic implications in section 4. Finally, we conclude with section 5.

2 R&D Activity, Free Revealing and Constant Cost Reductions

2.1 Basic Model

Consider an industry composed of two firms, firm 1 and firm 2, with identical initial unit costs c_0, competing in R&D in the same market for the same class of products. In our particular setup, the innovation race is modelled as a two-stage simultaneous move game, that is firms enter R&D competition for two periods. Time is discrete and at each date $t = (1, 2)$, firm i invests an amount $x_i^t \in [0; 1]$ at a quadratic cost $c(x_i^t) = \frac{1}{2} x_i^{t^2}$ in R&D in order to succeed in the innovation process. Both firms discount future expected payoffs at a rate $\delta = 1$. The instantaneous probability of success of firm i corresponds to its current R&D investment x_i^t (or its current R&D effort). Thus for each firm i the set of possible outcomes at time t is $Y_t = \{S_t, F_t\}$ with probabilities $\{x_i^t, 1 - x_i^t\}$, S stands for "success" and F for "failure"[3].

The joint outcomes at each date depends on the design of the instrument that promotes innovation. We assume that at the end of each period a regulator rewards the successful firm(s) with a patent, such that either one of the two firms wins the race, or both, or none of them[4]. More precisely, the

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[3] A patent race can also be of the Poisson type, along a continuous time space, with statistically independent successes for the two firms conditional on their knowledge levels (see e.g. Loury (1979), Dasgupta and Stiglitz (1980), Reinganum (1982)).

[4] Although the ex-aequo issue seems to be unusual, we justify this possibility by claiming that in the real economy, such a situation might occur, especially when one period is infinitely small. Effectively, it is well known that if the time space is continuous, the probability of success during a lack of time dt tends to zero for the two firms. Thus, if both firms have similar initial unit costs, then it is possible that both firms innovate or not before any date t.

rewarding process is conditional to the firms' ability to fulfill the conditions required by the regulator[5]. One particular condition might be the reaching of a specific level of unit cost c_t and we will concentrate on such a goal to reach for the firms[6]. That is why this scheme allows the time space to be discrete and this leads to the following set of possible joint outcomes $YY_t = \{SS_t, SF_t, FS_t, FF_t\}$ with probabilities $\{x_1^t x_2^t, x_1^t(1-x_2^t), (1-x_1^t)x_2^t, (1-x_1^t)(1-x_2^t)\}$[7]. For instance SF_t means that, at time t, firm 1 succeeds and reaches a unit cost c_t whereas firm 2 fails and keeps on producing with its previous unit cost. We should clarify that a firm succeeds only if it reaches the goal c_t at any time (from date t) before the opponent does so. For instance if both firms fail in the first period, then they will compete in period 2 for the first innovation that reduces the unit cost from c_0 to c_1.

We do not let the possibility of the following firm to leapfrog the leader, that means that the follower can at most catch up with but cannot overpass the leader. Particularly, a firm is not able to reach a unit cost c_2 in the second period if it has not reached a unit cost c_1 in the first period. Consequently, the following firm after the first round of competition, that is the firm who fails at $t = 1$ whereas the other succeeds, cannot be the leading firm after the next round at $t = 2$. We assume also that the length of a patent is infinite, which means that there is no way for the following firm to appropriate the technology of the leading firm after any period or put differently, the innovation never becomes a public good[8].

The two firms compete as a Bertrand duopoly, that is a firm gets a positive profit at each date only if it succeeds whereas the other firm fails. Formally, this situation occurs only if the issue YY_t belongs to the subset $\{SF_t, FS_t\}$ and the successful firm i earns a profit $\pi_{ji} = (c_j - c_i)\, D(c_j)$ whereas the other gets zero with probability $x_i^t(1-x_j^t)$. Finally, for simplicity we choose a unitary demand function. This simplifies the determination of the payoffs. Another interpretation is the presence of a unique consumer in the economy. Specifically, the demand function, that firm i faces, takes the values $D(c_j) = 1$ if $c_j > c_i$ and $D(c_j) = 0$ if $c_j \leq c_i$, such that if firm i is the leader it gets a positive profit $\pi_{ji} = (c_j - c_i)$. In addition, we allow the cost reduction π to be a constant from a period to the next, such that $c_0 - c_1 = c_1 - c_2 = \pi$.

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[5] Obviously the existence of the regulator who rewards the winner is implicit since the leading firm gets a larger profit than the follower.

[6] Thus our specific innovation promoting instrument is a policy mix between IP and a prize system: firms are rewarded with a patent conditional on a particular required innovation size (unit cost in our case).

[7] The implicit assumption of this probability distribution is that both individual probabilities of success are independent.

[8] Another way to justify the impossibility for the following firm to appropriate the leading technology is to assume that the imitation cost is equal or larger than the investment cost required to succeed.

As we study a dynamic situation we must clarify a particular point regarding the relative payoffs. After the first period of competition, if the outcome is asymmetric, then the follower will not earn more than zero profits by the end of the second stage. Effectively, if the follower fails once more in the second period, then it will remain the follower, but if it succeeds it will reach at most a unit cost c_1 whereas the other firm already reached that same unit cost in the first stage. So either the leader succeeds once more at $t = 2$ or fails, and the follower gets zero demand or half of the total demand, such that in every scenario the payoff to the following firm is equal to zero. Thus, after the first stage the following firm drops out of the R&D race and the leader charges a price equal to the unit cost of its opponent c_0, while it appropriates the whole demand. Formally, when YY_1 belongs to $\{SF_1, FS_1\}$, the profit of the leader in the second period is equal to $\pi_{02} = (c_0 - c_2) = 2\pi$ if it succeeds with probability x_i^2 or $\pi_{01} = (c_0 - c_1) = \pi$ if it fails with probability $1 - x_i^2$, and the payoff to the follower is equal to zero in every case.

The losing firm after one period stops its R&D activity but remains in the market, so that the leader is not able to charge a monopoly price but engages in Bertrand duopoly pricing. In fact this behavior depends on the nature of the leading innovation, that is if it is drastic, which corresponds to the case where the potential monopoly price is lower than the marginal cost of the opponent (the follower), or not. If it was, then the leading firm could effectively charge a monopoly price. Consequently, our assumption on the behavior of the leading firm means that the innovation is not drastic and that the leader has to take into account the presence of the follower, such that we still have a Bertrand competition. Finally, figure 1 synthesizes the distribution of the payoffs over the whole game.

The equilibrium concept is a perfect Nash equilibrium in investment strategies over both periods $\{x_i^1, x_i^2\}, i = (1, 2)$. As the model refers to a two-stage simultaneous move game, we have the possibility to solve the problem by backward induction. The firms invest in the first period and observe the outcome of the first stage before they choose what amount to invest in the second period.

First, we have to characterize the continuation value functions (CVFs) after each state of the first stage. Obviously, as there are four possible outcomes after one period ($YY_1 = \{SS_1, SF_1, FS_1, FF_1\}$), we have four CVFs for every firm, which we denote by $V_{i_{YY}} = \{V_{i_{SS}}, V_{i_{SF}}, V_{i_{FS}}, V_{i_{FF}}\}$. As the potential cost reduction remains the same after any symmetric outcome $YY_1 = \{SS_1, FF_1\}$ in the first stage, the CVFs relative to those two states are also equal. This involves the consideration of three distinct cases according to the relative position of each firm after the first round: either the two firms are ex-aequo (equality of the unit costs), or a firm leads (smaller unit cost), or that same firm follows (larger unit cost). Thus the corresponding notations are given by

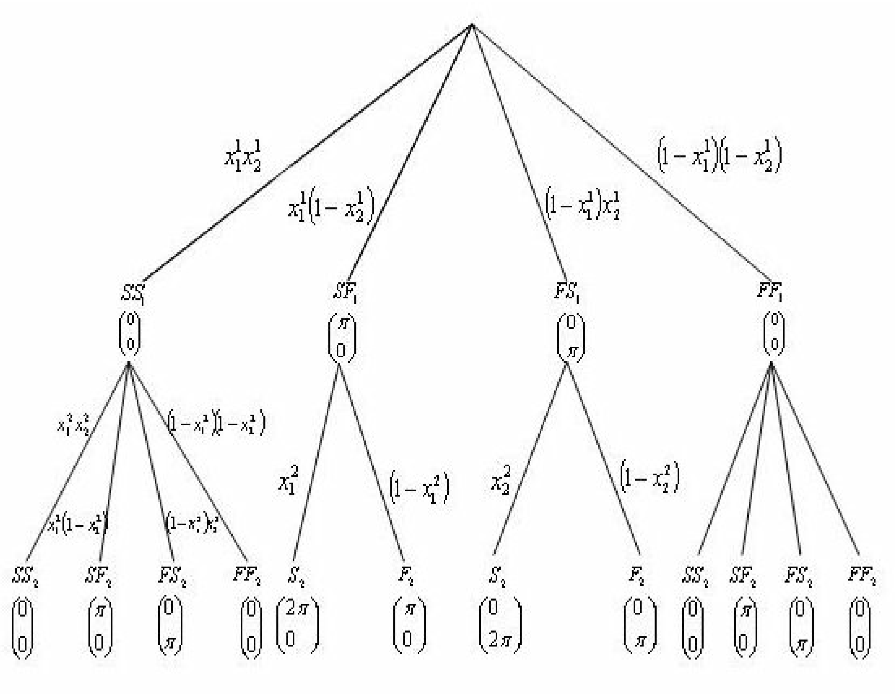

Figure 1: Game Tree of the Basic Model

$Y = \{E, S, F\}$ and $V_Y = \{V_E, V_S, V_F\}$. Thesec expressions are given by:

$$V_E = \max_{x_i^2} \left\{ x_i^2(1 - x_j^2)\pi - \frac{1}{2} x_i^{2^2} \right\} \tag{1}$$

$$V_S = \max_{x_i^2} \left\{ x_i^2 2\pi + (1 - x_i^2)\pi - \frac{1}{2} x_i^{2^2} \right\} \tag{2}$$

$$V_F = 0 \tag{3}$$

To solve the game, we proceed as follows: first, maximize each of these functions with respect to the relative firm's second stage investment strategy, get the best response functions (BRFs) to the opponent's choice. Second, find the intersection between each pair of BRFs corresponding to each outcome of the first stage in order to determine the optimal levels of investment in the second period and finally, compute each CVF. Thus, the basic maximization problem yields the following BRFs which are denoted by $x_{i_Y}^2 \left(x_{j_Y}^2 \right)$:

$$x_{i_E}^2 \left(x_{j_E}^2 \right) = \left(1 - x_{j_E}^2 \right) \pi \tag{4}$$

$$x_{i_S}^2 \left(x_{j_F}^2 \right) = \pi \tag{5}$$

$$x_{i_F}^2\left(x_{j_S}^2\right) = 0 \tag{6}$$

As described above, the optimal levels of second stage investments relative to each state after one period of competition $(x_{i_Y}^{2^*})$ correspond to the intersection point between $x_{i_Y}^2\left(x_{j_Y}^2\right)$ and $x_{j_Y}^{2^{-1}}\left(x_{i_Y}^2\right)$. After an asymmetric outcome, the best response function of each firm is also the optimal level of investment since only one firm, the leader, remains in the second stage race. Since firms are identical we can easily show that $x_{i_E}^{2^*} = x_{j_E}^{2^*} = x_E^{2^*}$, that is the subgame has a symmetric equilibrium. The equilibrium values that characterize the perfect Nash equilibrium in the second stage are for $i = (1, 2)$:

$$x_E^{2^*} = \frac{\pi}{1+\pi} \tag{7}$$

$$x_S^{2^*} = \pi \tag{8}$$

$$x_F^{2^*} = 0 \tag{9}$$

A first result states that a firm is more willing to invest when it is the leader after one shot, and thus is the unique remaining firm in the R&D activity, than after any symmetric outcome $YY_1 = \{SS_1, FF_1\}$. This makes sense given that a leading firm is ensured to make a positive profit by the end of the second period, even if it fails whereas, after a symmetric outcome, there is a great probability of getting zero profit. We can interpret this behavior as a characteristic of the risk aversion of the firms who are afraid of loosing their investments. Furthermore, equation 8 provides a constraint on the instantaneous profit that has to belong to the interval $[0; 1]$. Now we can compute the continuation values V_Y.

$$V_E = \frac{\pi^2}{2\left(1+\pi\right)^2} \tag{10}$$

$$V_S = \frac{\pi^2}{2} + \pi \tag{11}$$

$$V_F = 0 \tag{12}$$

The solution to the first stage problem is then solved by backward induction. Since we are able to find an interior solution to the second stage problem, we can find, using the same procedure, the solution to the first stage problem and thus characterize a perfect Nash equilibrium in investment strategies over both periods. The value functions for firm i at $t = 0$ is given by

$$\begin{aligned} V_i &= \max_{x_i^1}\{x_i^1 x_j^1 V_E + x_i^1\left(1 - x_j^1\right)\left(V_S + \pi\right) + \\ &\quad + \left(1 - x_i^1\right)\left(1 - x_j^1\right) V_E - \frac{1}{2} x_i^{1^2}\} \end{aligned} \tag{13}$$

Similarly to the second stage problem solved above, we can find the best response functions $x_i^1\left(x_j^1\right)$ as well as the equilibrium mixed strategies $x_i^{1^*}$.

$$x_i^1\left(x_j^1\right) = \left[(2V_E - V_S) - \pi\right] x_j^1 + \left[(V_S - V_E) + \pi\right] \tag{14}$$

The symmetric equilibrium in investment strategies in th first perod are $(x_i^{1^*} = x_j^{1^*} = x^{1^*})$

$$x^{1^*} = \frac{V_S - V_E + \pi}{1 + (V_S - 2V_E + \pi)} \tag{15}$$

Thus, we have identified the complete perfect Nash equilibrium in investment strategies over two stages of R&D competition, which calls forth the following proposition:

Proposition 1 *In our two stage dynamic model of R&D competition with competitive behavior and constant cost reductions, the perfect Nash equilibrium is characterized by the strategies played by firm i* $x^{1^*} \in [0;1]$ *in the first stage and* $x_Y^{2^*} \in [0;1]$ *,according to firm i's first period relative position* $Y \in \{E, S, F\}$*, in the second stage,* $\forall \pi \in [0;1]$. *(proof, see Appendix)*

2.2 Knowledge Disclosure

This section analyze the effects of knowledge disclosure at the end of the first stage on the investment behaviors of the firms in this specific framework. By knowledge disclosure we mean an information sharing through a direct bargaining mechanism from an informed firm to the uninformed firm. Obviously, knowledge disclosure occurs only right after an asymmetric outcome has been reached. Moreover, a leading firm is not able to pretend to have less knowledge than it has, in order to avoid information sharing that is enforced by the contract. This potential moral hazard is screened by the specific scheme we consider in this analysis, because the Bertrand competition immediately reveals the amount of knowledge of the two firms[9].

In this subsection, we will focus on a particular case of information sharing, that is the case where both firms contract to freely reveal their information. This corresponds to a zero license fee contract. Such a contract can take place since the two firms are identical and our previous results showed us that both firms face the same probability of being the leader (the follower) after the first period. Thus, the expected profits at $t = 0$ are the same amongst the firms

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[9] We could discuss this underlying assumption of ex-post observability, but this is not the main task of that paper. As mentionned by Scotchmer and Gallini (2000), the ex-post observability might play a substantial role in the effectiveness of an innovation promoting instrument (such as IP or a prize system). Thus, the relaxation of this assumption could involve different results in further research.

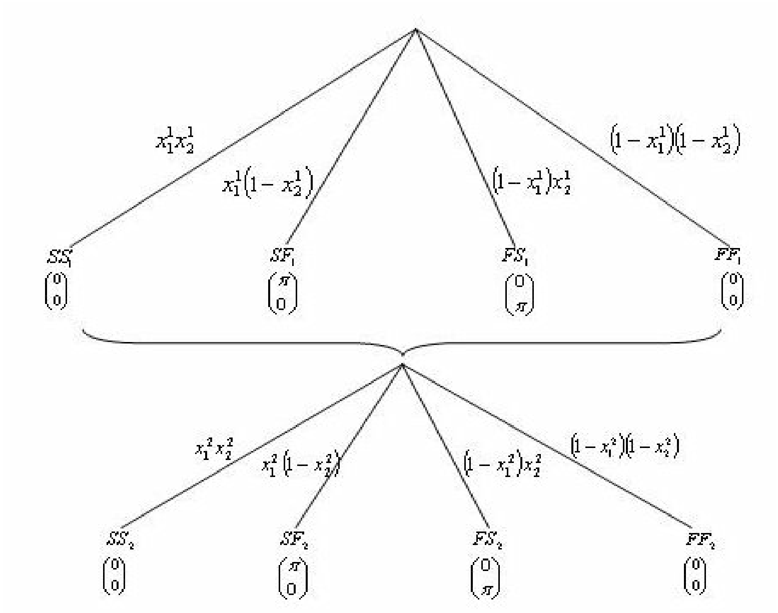

Figure 2: Game Tree of the Disclosure Case

whatever the amount of the fee that has to be paid. That is why we allow the fee to be equal to zero.

Moreover, since we have Bertrand competition, the following firm is going to sign the contract only if there is full disclosure from the leading firm. Effectively, recalling the assumption that a following firm cannot leapfrog the leader, so that if the contract (assumed perfectly enforceable) does not involve an equality of the unit costs between the two firms after the first round of competition, the follower will not even have the chance to catch up with the leader in the next period. Thus, it has no incentive to sign such a contract. The only contract that is accepted by a follower is a full disclosure contract.

Finally, the bargaining mechanism and the signature of the contract have to take place before the R&D activity at $t = 0$. Obviously, a leading firm does not have any incentives to disclose any amount of knowledge at $t = 1$ since $V_S > V_E$. On the other hand, firms know at $t = 0$ that there is a non zero probability of being the following firm at $t = 1$, so that there might be an incentive to sign the contract. That is why the direct bargaining mechanism and the license fee contract have to be ex-ante to the R&D activity.

If we refer to the payoffs distribution represented in figure 2, we can write the continuation values as below (the superscript D is for *Disclosure*):

$$V_E^D = V_E = \frac{\pi^2}{2\left(1+\pi\right)^2} \tag{16}$$

$$V_S^D = V_F^D = V_E \tag{17}$$

Using the same procedure than in the basic model, we can also show that the investment strategy in the second period is homogeneous along the different possible outcomes at $t = 2$, that is a firm invests always the same amount in the second stage, whatever the knot that has been reached at the end of the first period. Obviously, this is due to both the disclosure behavior and the constant cost reduction π along the time space. Furthermore, that second stage equilibrium strategy is identical to the one played in the second period of the basic model after a symmetric outcome (see equation 7) and is equal to

$$x_E^{2^*} = \frac{\pi}{1+\pi} \tag{18}$$

If we solve the first stage problem and compute the equilibrium strategies for $i = (1, 2)$, we obtain an interesting result: the optimal investment strategy in the first period is equal to the one in the second period. As the two firms are identical and since the disclosure behavior involves, at each period, an equality of the unit costs, the game in the disclosure case takes the form of a repeated game with identical expected payoffs at each date. Thus, it is not really surprising to get such a result. Formally, the maximization problem of the first stage and the resulting equilibrium strategy yields:

$$V_i^D = \max_{x_i^1}\{V_E + x_i^1\left(1 - x_j^1\right)\pi - \frac{1}{2}x_i^{1^2}\} \tag{19}$$

$$x_i^{1^{D*}} = \frac{\pi}{1+\pi} = x^{D*} \tag{20}$$

Proposition 2 *In our two stage dynamic model of R&D competition with knowledge disclosure and constant cost reductions, the perfect Nash equilibrium is characterized by a constant investment strategy* $x^{D*} \in [0; 1]$ *played by firm i at each period of the game,* $\forall \pi \in [0; 1]$. *(the proof is similar to proposition 1)*

2.3 Behavior Decisions Analysis

Does there exist a range of values for the parameter π such that disclosure is profitable? We denote the optimal value functions by $V(\pi)$ and $V^D(\pi)$ for the competitive and the disclosure behaviors. Given the different forms of the payoffs in the basic model according to the nature of an outcome at the end of the first period (symmetric or asymmetric), the corresponding value function $V(\pi)$ cannot be expressed in an easy manner, whereas $V^D(\pi)$ is simply equal to $2V_E$. Thus the best way to show the eventual existence of an interval for π such that $V(\pi) - V^D(\pi) \leq 0$ is a plot of the difference between those two

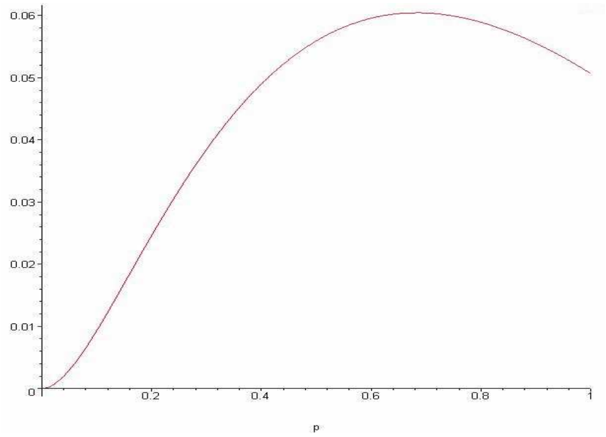

Figure 3: Difference of the Value Functions

value functions as a function of the instantaneous profit π. We obtain the graph represented in figure 3[10].

Then the main result of our basic framework is straight forward. The unique value for which the disclosure behavior involves a value function not smaller than the competitive behavior is $\pi = 0$ and both value functions as well as the investment strategies are equal to zero. This leads to proposition 3.

Proposition 3 $V(\pi) - V^D(\pi) \geq 0, \forall \pi \in [0;1]$*, with strict inequality when* $\pi > 0$*. (proof, see Appendix)*

The intuition behind this result is the destruction of the rents. If the firms behave competitively, then each firm can earn, at $t = 2$, either 0, π or 2π. On the contrary, if the firms choose to disclose their knowledge, the highest possible payoff at the end of the second period is π. Consequently, information sharing destroys rents. Another explanation to understand this is that the firms evaluate the CVF after a symmetric issue at the corresponding investment cost (see that $V_E = c(x_E^{2^*}))$, whereas the CVF after the *S-state* is larger than the corresponding investment cost ($V_S = c(x_S^{2^*}) + \pi$). Thus, even if knowledge disclosure decreases the R&D effort[11], by doing so, the firms strictly evaluate the innovation project at its investment cost. If the firms do not share their information, then the project is evaluated at a greater value than its investment cost. This is true for any $\pi \in [0;1]$, so that the firms never reveal their information when the cost reductions are constant.

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[10] In figure 3, the instantaneous profit π is denoted by p.

[11] We can easily show that $x^{1^*} \geq x^{D^*}, \forall \pi \in [0;1]$.

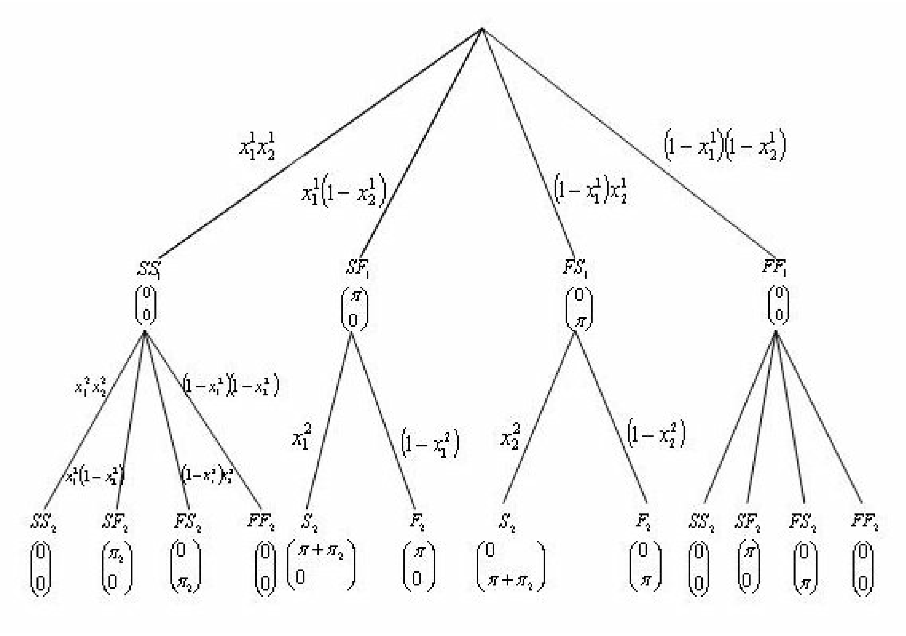

Figure 4: Game Tree with Progressive Cost Reductions

3 R&D Activity, Free Revealing and Progressive Cost Reductions

3.1 Competitive Behavior

In this section, we relax the assumption of constant cost reductions. From now on the cost reduction involved by the second invention π_2 is different from the cost reduction involved by the first invention, that is $\pi_2 = (c_1 - c_2) \neq \pi = (c_0 - c_1)$. Thus, using analogous methods to those used in the basic model, we can characterize the equilibrium strategies as well as the CVFs. Figure 4 represents the game tree as well as the distribution of the payoffs. The subscript PC is for *progressive cost* reductions.

$$V_{SS_{PC}} = \max_{x_i^2} \left\{ x_i^2(1-x_j^2)\pi_2 - \frac{1}{2}x_i^{2^2} \right\} = \frac{\pi_2^2}{2\left(1+\pi_2\right)^2} \tag{21}$$

$$V_{FF_{PC}} = \max_{x_i^2} \left\{ x_i^2(1-x_j^2)\pi - \frac{1}{2}x_i^{2^2} \right\} = \frac{\pi^2}{2\left(1+\pi\right)^2} = V_E \tag{22}$$

$$V_{S_{PC}} = \max_{x_i^2} \left\{ x_i^2\left(\pi+\pi_2\right) + (1-x_i^2)\pi - \frac{1}{2}x_i^{2^2} \right\} = \frac{\pi_2^2}{2} + \pi \tag{23}$$

$$V_{F_{PC}} = 0 \tag{24}$$

The corresponding equilibrium strategies are for any firm i, $i = (1, 2)$

$$x^{2^*}_{SS_{PC}} = \frac{\pi_2}{1+\pi_2} \tag{25}$$

$$x^{2^*}_{FF_{PC}} = \frac{\pi}{1+\pi} = x^{2^*}_{E} \tag{26}$$

$$x^{2^*}_{S_{PC}} = \pi_2 \tag{27}$$

$$x^{2^*}_{F_{PC}} = 0 \tag{28}$$

Those equilibrium values (particularly equation 27) provide a constraint on the parameter π_2. Effectively, each investment strategy has to belong to the interval $[0;1]$ such that $\pi_2 \in [0;1]$. Moreover, the CVFs after the two symmetric outcomes are not equal anymore. Now we can characterize the value function at $t = 0$ as well as the equilibrium investment strategy $x^{1^*}_{PC}$.

$$\begin{aligned} V_{i_{PC}} &= \max_{x^1_i}\{x^1_i x^1_j V_{SS_{PC}} + x^1_i \left(1-x^1_j\right)\left(V_{S_{PC}}+\pi\right) + \\ &\quad + \left(1-x^1_i\right)\left(1-x^1_j\right) V_{FF_{PC}} - \frac{1}{2} x^{1^2}_i\} \end{aligned} \tag{29}$$

$$x^{1^*}_{PC} = \frac{V_{S_{PC}} - V_{FF_{PC}} + \pi}{1 + V_{S_{PC}} - V_{SS_{PC}} - V_{FF_{PC}} + \pi} \tag{30}$$

Using an appropriate software, since we know all the parameters that enter those two expressions, we can easily compute them. However, the final expression of the value function is a polynomial of very high degrees for π and π_2. For the ease of exposition, we will present the final solution graphically.

For further discussion, we show that $x^{1^*}_{PC}$ is a non decreasing function in π and π_2, so that there is a backward spillover from the second innovation payoff to the first one. Moreover, we have to check that $x^{1^*}_{PC}$ belongs to the interval $[0;1]$. Proposition 4 is derived.

Proposition 4 $x^{1^*}_{PC} \in [0;1]$. $x^{1^*}_{PC}$ *is a non decreasing function in* π *and* $\pi_2, \forall \pi \geq 0$ *and* $\pi_2 \in [0;1]$. *(proof, see Appendix)*

3.2 Knowledge Disclosure

Following the same reasoning described in the corresponding subsection of the model with constant cost reductions, we let the possibility to the firms to contract, ex-ante to the first period, an agreement about the sharing of their information. As we have seen in subsection 3.1, the CVF after $YY_1 = FF_1$ is different from the CVF after $YY_1 = SS_1$. The distribution of the payoffs when the firms freely disclose their information is summarized in figure 5.

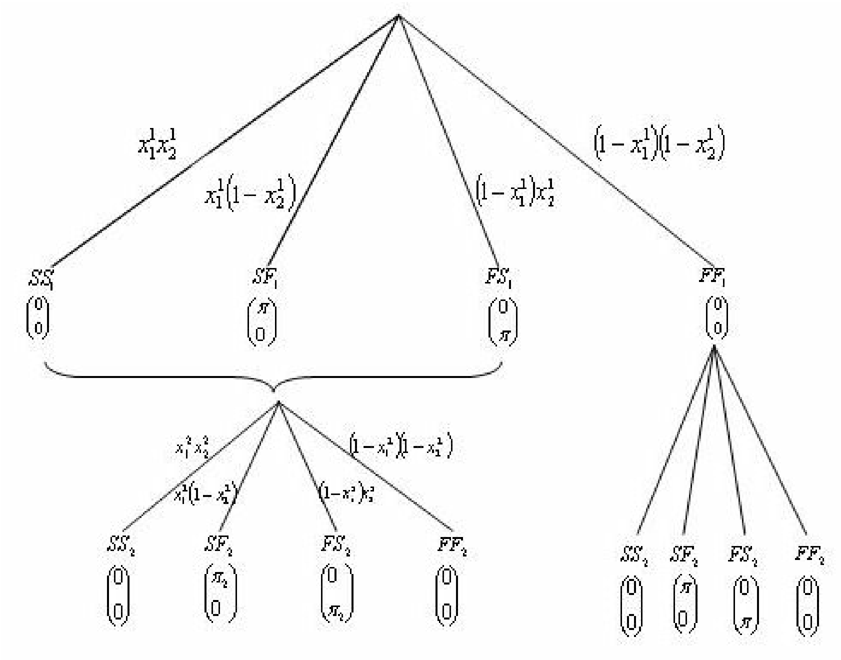

Figure 5: Game Tree with Progressive Cost Reductions and Disclosure

The equilibrium strategies and CVFs relative to the SS and FF states are identical to the corresponding ones in the previous model with competitive behavior and progressive cost reductions, and $V_{S_{PC}}^{D}=V_{F_{PC}}^{D}=V_{SS_{PC}}$. We only present in this subsection the optimal level of investment of the first stage and the implicit value function at $t=0$. A firm solves the following maximization problem

$$\begin{aligned} V_{i_{PC}}^{D} &= \max_{x_i^1}\{x_i^1 x_j^1 V_{SS_{PC}} + x_i^1\left(1-x_j^1\right)\left(V_{SS_{PC}}+\pi\right)+ \\ &\quad + \left(1-x_i^1\right)x_j^1 V_{SS_{PC}} + \left(1-x_i^1\right)\left(1-x_j^1\right)V_{FF_{PC}} - \frac{1}{2}x_i^{1^2}\} \end{aligned} \tag{31}$$

Then, the symmetric solution that constitutes the perfect Nash equilibrium in investment strategies in the first period is

$$x_{PC}^{1^{D*}}=\frac{V_{SS_{PC}}-V_{FF_{PC}}+\pi}{1+V_{SS_{PC}}-V_{FF_{PC}}+\pi} \tag{32}$$

Relative to the previous subsection, we can also show that $x_{PC}^{1^{D*}}$ belongs to the interval $[0;1]$ and is also a non decreasing function in π and π_2.

Proposition 5 $x_{PC}^{1^{D*}}\in[0;1]$. $x_{PC}^{1^{D*}}$ *is a non decreasing function in* π *and* $\pi_2, \forall\pi\geq 0$ *and* $\pi_2\in[0;1]$. *(the proof is similar to proposition 4)*

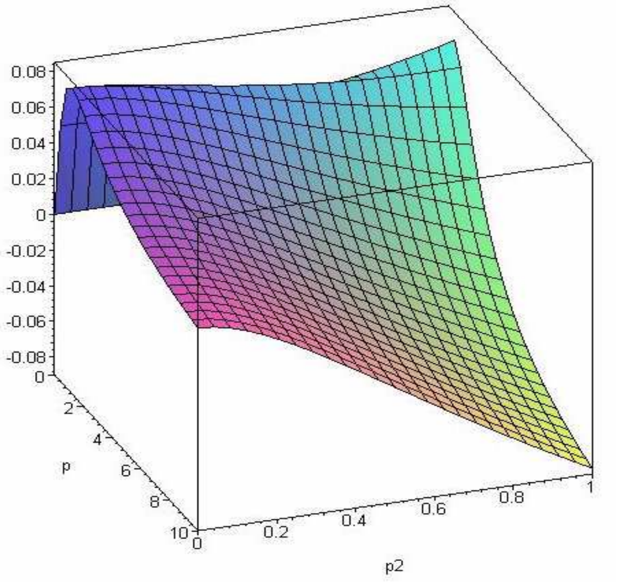

Figure 6: Difference of the Value Functions $V_{PC}\left(\pi,\pi_2\right)-V_{PC}^{D}\left(\pi,\pi_2\right)$

3.3 Behavior Decisions Analysis

This analysis should give us the opportunity to answer whether there exists any ranges of values for π and π_2 such that the full disclosure behavior involves a higher value function than the competitive behavior. We will focus on the difference between the two value functions V_{PC} and V_{PC}^{D} as a function of π and π_2. As we noticed above, the explicit forms of the two value functions are to complicated so that the comparative statics can be done analytically in a rigorous manner. Using again the same software, we can plot the difference of those two value functions in a 3-dimensional graph in $\left(\pi,\pi_2,V_{PC}\left(\pi,\pi_2\right)-V_{PC}^{D}\left(\pi,\pi_2\right)\right)$ and check if the disclosure case involves higher expected profits so that $V_{PC}\left(\pi,\pi_2\right)-V_{PC}^{D}\left(\pi,\pi_2\right)<0$. Figure 6 shows an overview of the so called difference where $\pi>0$ and $\pi_2\in[0;1]$, whereas figure 7 plots the same difference of value functions but for negative values[12].

The second graph shows clearly the existence of situations where firms have incentives to cooperate by revealing their information after the first stage. Particularly, if the profit of the first invention is large, as well as π_2, both firms prefer to share their knowledge. Moreover we can show that this behavior also occurs when both instantaneous profits are small. However, disclosure is more likely to occur if both payoffs are large. We synthesize our results in propositions 6 and 7.

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[12] In those graphs π is denoted by p and π_2 by $p2$.

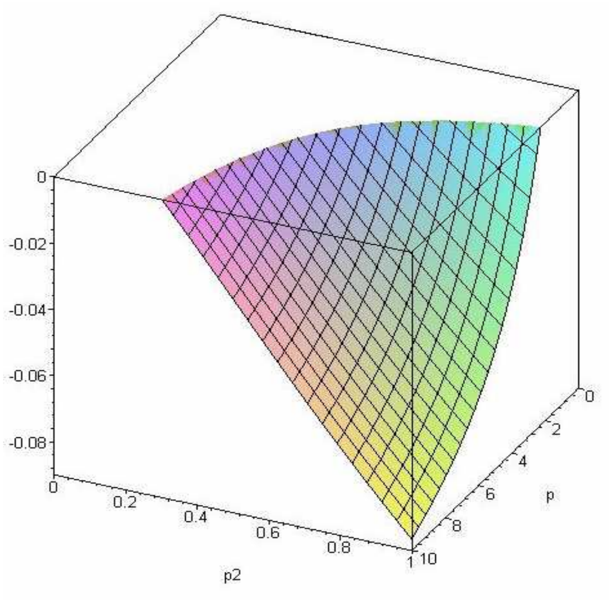

Figure 7: Difference of the Value Functions $V_{PC}(\pi, \pi_2) - V_{PC}^{D}(\pi, \pi_2) < 0$

Proposition 6 *i) If* π *tends to* ∞*, then* $V_{PC}^{D}(\pi, \pi_2) \geq V_{PC}(\pi, \pi_2)$*,* $\forall \pi_2 \in [0; 1]$*. And ii) if* π_2 *tends to* 1*, then* $\exists \pi_0 > 0$ *such that if* $\pi \geq \pi_0$ *then* $V_{PC}^{D}(\pi, \pi_2) \geq V_{PC}(\pi, \pi_2)$*. (proof, see Appendix)*

Proposition 7 *If* π *tends to* 0*, then* $\exists \pi_{2_0} > 0$ *such that if* $\pi_2 \leq \pi_{2_0}$ *then* $V_{PC}^{D}(\pi, \pi_2) \geq V_{PC}(\pi, \pi_2)$*. (proof, see Appendix)*

We need to formalize and to prove another important result. The investment strategy in the first stage is always smaller under disclosure behavior than under competitive behavior. The investment costs are also smaller when they reveal their information.

Proposition 8 $x_{PC}^{1^*} \geq x_{PC}^{1^{D*}}, \forall \pi \geq 0$ *and* $\pi_2 \in [0; 1]$*. (proof, see Appendix)*

We show that the *SS-state* is reached with an equal or higher probability when firms share their information than when they do not so. This probability plays a substantial role in the behavior decisions of the firms. To see that, we compute the difference of the indirect value functions under disclosure and competitive behavior.

$$\begin{aligned}
V_{PC}^{D}\left(x_{PC}^{1^{D*}}\right)-V_{PC}\left(x_{PC}^{1^{*}}\right) &= \left[\left(x_{PC}^{1^{D*}}\right)^{2}+2x_{PC}^{1^{D*}}\left(1-x_{PC}^{1^{D*}}\right)-\left(x_{PC}^{1^{*}}\right)^{2}\right]V_{SS_{PC}}+\\
&+\left[\left(1-x_{PC}^{1^{D*}}\right)^{2}-\left(1-x_{PC}^{1^{*}}\right)^{2}\right]V_{FF_{PC}}+\\
&+\left[x_{PC}^{1^{D*}}\left(1-x_{PC}^{1^{D*}}\right)-x_{PC}^{1^{*}}\left(1-x_{PC}^{1^{*}}\right)\right]\pi-\\
&-x_{PC}^{1^{*}}\left(1-x_{PC}^{1^{*}}\right)V_{S_{PC}}-\frac{1}{2}\left[\left(x_{PC}^{1^{D*}}\right)^{2}-\left(x_{PC}^{1^{*}}\right)^{2}\right]
\end{aligned}$$

The difference between the probabilities that the *SS-state* occurs under disclosure behavior and competitive behavior by

$$\begin{aligned}
\triangle\Pr SS &= \Pr(SS-state \mid disclosure)-\Pr\left(SS-state \mid competitive\right) \qquad (33)\\
&= \left[\left(x_{PC}^{1^{D*}}\right)^{2}+2x_{PC}^{1^{D*}}\left(1-x_{PC}^{1^{D*}}\right)\right]-\left[\left(x_{PC}^{1^{*}}\right)^{2}\right]
\end{aligned}$$

Our results call forth the last proposition

Proposition 9 $\triangle\Pr SS \geq 0, \forall\pi \geq 0$ *and* $\pi_2 \in [0;1]$. *(once* $\triangle\Pr SS$ *is computed, the proof is similar to proposition 3)*

However, we have shown that the firms share their information either when π and π_2 are large, or when both are small. According to figure 8, we see that disclosure effectively occurs when $\triangle\Pr SS$ is close to zero[13].Similarly, when π and π_2 are large or small, the probability of earning a payoff π at the end of the first stage under disclosure behavior is close to the same probability when firms do not share their information[14], and firms decide to disclose their knowledge. The difference of the probabilities of getting π after one round of competition is given by:

$$\begin{aligned}
\Delta\Pr\pi &= \Pr(\pi \mid disclosure)-\Pr\left(\pi \mid competitive\right)=\\
&= \left[x_{PC}^{1^{D*}}\left(1-x_{PC}^{1^{D*}}\right)-x_{PC}^{1^{*}}\left(1-x_{PC}^{1^{*}}\right)\right]
\end{aligned}$$

---

[13] We proove this by showing that, on the one hand

$$\lim_{\substack{\pi\to 0\\ \pi_2\to 0}}\triangle\Pr SS=0$$

and on the other hand

$$\lim_{\pi\to\infty}\triangle\Pr SS=0$$

[14] The proof is similar to the one used with $\Delta\Pr SS$.

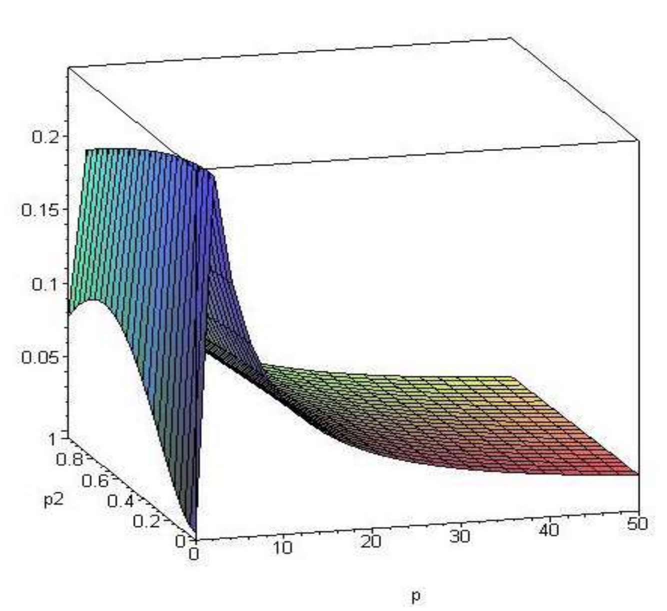

Figure 8: Plot of $\triangle \Pr SS$ as a function of π (p-axis) and π_2 (p2-axis).

Figure 9 plots this expression.

4 Discussion

Firms disclose their knowledge either when both instantaneous payoffs are small or when they are large. Furthermore, propositions 6 and 7, as well as figure 7, provide us a great intuition so that there might effectively exist intervals for π and π_2 such that the disclosure behavior involves higher expected profits than the competitive behavior. Unfortunately, given the form of the value functions, we are able to prove that result only for extreme values of π and π_2. However we provide some intuitive explanations of those results.

First, we point out what is really relevant to the disclosure behavior in this study. A priori, firms do not have strong incentives to reveal their information because the ultimate outcome to reach for a firm is to succeed in both periods, hoping that the other firm fails in the first stage (and thus in the second stage too), so that the profit is maximized. But firms know there is a possibility of being the follower after the first round. Thus, what leads the firms to disclose their knowledge is not an eventual altruistic feeling, but the sign of a contract, which we assumed perfectly enforceable, before the R&D activity takes place. Effectively the most attractive aspect of this direct bargaining mechanism is the eventuality of getting the information for free if you are the follower, and thus, remaining in the race in the second period. And since there exists a non

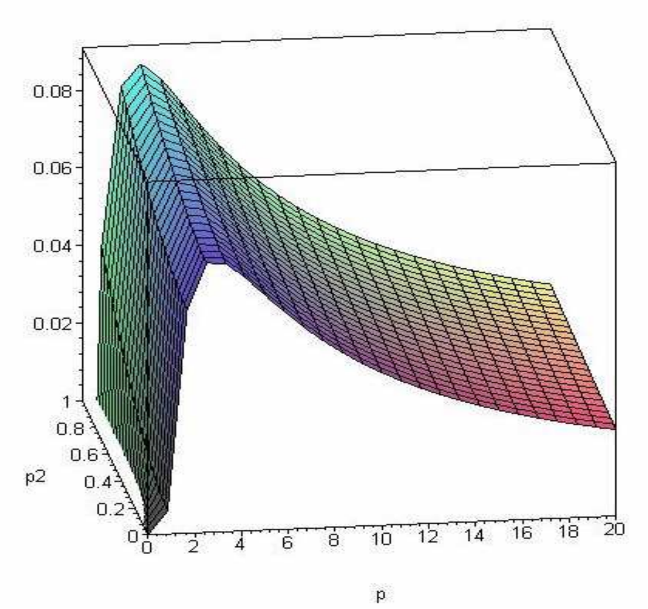

Figure 9: Plot of $\triangle \Pr \pi$ as a function of π (p-axis) and π_2 (p2-axis).

zero probability of being behind at $t = 1$, the firms might have incentives to sign such a contract.

Second, we discuss the case where π and π_2 are large. We know that the investment strategy under competitive behavior in the first period is an increasing function in both π and π_2 (see proof of proposition 4). Thus, when π and π_2 are large, each firm suspects its opponent to increase its investment in order to reach a high probability of success. Consequently, as none of the two firms wants to be the follower at the second stage, we can be sure that both firms will invest large amounts of money in R&D, so that they will reach the *SS-state* after the first stage with a high joint probability. Knowing this, the firms choose to disclose their information in order to increase or to keep constant the occurrence of that state. Despite the reduction of the equilibrium investment strategies in the first period, the *SS-state* is reached with a higher or equal probability under knowledge disclosure than under competitive behavior. More precisely, firms choose their investment strategies in the first period when there is information sharing, such that both probabilities of being in the *SS-state*, in the disclosure case and in the competitive case, are close. Effectively, the firms invest as little as possible, so that both keep on their R&D activity for the second innovation with the same probability as in the no disclosure case. Thus, If the firms disclose their knowledge, on the one hand, they insure themselves of remaining in the second stage and, on the other hand, they will compete for the second innovation (which yields a large profit π_2) in the second period with a higher but close probability to the one when they do not reveal their information. Furthermore, the probability of getting a large profit π at the end of the first period, when firms decide to disclose, is also higher or equal to that same probability when firms do not reveal their information. However, the main benefit from disclosing is that they substantially decrease their investment costs, whereas both probabilities of competing for the second innovation and earning a large profit π at $t = 1$ remain the same.

Third, we explain proposition 7. When π tends to zero and π_2 is small, both firms invest small amounts of money, such that the probability of being in the *FF-state* at the end of the first stage is high. We precise that a firm is not willing to invest a little bit more, such that it increases its chance to be the leader at $t = 1$, because such a strategy does not fit our perfect Nash equilibrium concept. However if the firms reach the *FF-state*, they will compete for a potential positive profit π that is close to zero, which seems not really attractive for the firms. Effectively, the firms will not invest in the second period in order to earn almost nothing. Thus, there are multiple benefits of a disclosure behavior. Once again, by sharing their information, the firms decrease their investment cost. Furthermore the joint probability of being in the *SS-state* increases or, at least, remains the same, so that the competition for a non zero positive profit π_2 has a greater or equal chance to occur. By the way, if the firms disclose their knowledge, they also avoid the *F-state.*

Finally, the main difference between propositions 6 and 7 is the following. In the first case, when π and π_2 are large, the firms disclose their knowledge especially in order to decrease the investment costs. Whereas in the second case, when π and π_2 are small, they choose to reveal their information because they prefer to compete for a non zero positive profit instead of competing for almost nothing.

5 Conclusion

Most of the papers relative to the patent system and R&D competition focus on the unique competitive behavior of the firms, whereas we showed that it may be profitable for the firms to reveal their information. We studied two cases, different with respect to the assumption on the instantaneous profits involved by the two innovations. First, if the cost reduction is constant in both periods, then the firms never agree to sign a contract before the R&D activity, such that they engage to reveal for free their information after the first round of competition. Second, assuming the cost reductions to be progressive, we showed analytically and graphically that disclosure occurs for extreme values of the parameters π and π_2. Moreover, the graphical representations of the difference between the value functions sustain that the firms choose to share their information for large intervals of those same parameters. Particularly, if π and π_2 are sufficiently large, the two firms disclose their knowledge in order to insure themselves of remaining in the race in the second period and, thus, avoid the possibility of being behind. Also, if π and π_2 are sufficiently small ($\pi < \pi_2$ and π is close to zero), information sharing occurs, because it is obviously more advantageous for the firms to compete for a non zero positive profit than for a very-close-to-zero profit. Finally, disclosure behaviors involve smaller investment costs in every case.

6 Appendix

Proof of proposition 1. As we show the existence of the perfect Nash equilibrium in the core text, we only prove here that $x^{1^*} = \frac{V_S - V_E + \pi}{1 + (V_S - 2V_E + \pi)} \in [0;1], \forall \pi \in [0;1]$. We simply show that x^{1^*} is an increasing function in π between 0 and 1, and that $x^{1^*}(\pi = 0) \geq 0$ and $x^{1^*}(\pi = 1) \leq 1$. But first we rewrite the expression of x^{1^*} as follows.

$$x^{1^*} = \frac{\pi\left(\pi^3 + 6\pi^2 + 8\pi + 4\right)}{\pi^4 + 6\pi^3 + +9\pi^2 + 8\pi + 2}$$

Thus, the derivative of x^{1^*} with respect to π is equal to :

$$\frac{\partial x^{1^*}}{\partial \pi} = \frac{2\pi^5 + 18\pi^4 + 56\pi^3 + 64\pi^2 + 32\pi}{\left(\pi^4 + 6\pi^3 + +9\pi^2 + 8\pi + 2\right)^2}$$

Since both the numerator and the denominator of this last expression are sums of positive terms for $\pi \in [0;1]$, $\frac{\partial x^{1^*}}{\partial \pi} \geq 0, \forall \pi \in [0;1]$. So x^{1^*} is a non decreasing function in π between 0 and 1. Finally $x^{1^*}(\pi = 0) = 0 \geq 0$ and $x^{1^*}(\pi = 1) = 19/26 \leq 1 \Rightarrow x^{1^*} \in [0;1], \forall \pi \in [0;1]$. ■

Proof of proposition 3. *i)* We show that $V(\pi) - V^D(\pi) > 0, \forall \pi \in]0;1]$. The explicit expression for the difference of the value function is:

$$\begin{aligned} V(\pi) - V^D(\pi) &= \frac{0.5\pi^{2(}12 + 56\pi + 108\pi^2 + 112\pi^3 + 45\pi^4 -}{\left(2 + 8\pi + 9\pi^2 + 6\pi^3 + \pi^4\right)(1+\pi)^2} \\ &\quad \underline{-18\pi^5 - 30\pi^6 - 10\pi^7 - \pi^8)} \\ &= \frac{0.5\pi^2 A}{B} \end{aligned}$$

Since $B > 0, \forall \pi \in]0;1]$, if $A > 0, \forall \pi \in]0;1]$ then $V(\pi) - V^D(\pi) > 0, \forall \pi \in]0;1]$. Let's rewrite A as follows

$$\begin{aligned} A &= 12 + \left(56\pi - 18\pi^5\right) + \left(108\pi^2 - 30\pi^6\right) + \left(112\pi^3 - 10\pi^7\right) + \\ &\quad + \left(45\pi^4 - \pi^8\right) \\ &= 12 + \sum_{i=1}^{4} k_i \end{aligned}$$

where $k_i = \gamma_i \pi^{m_i} - \delta_i \pi^{n_i}$. If $\pi \in]0;1]$, then $\gamma_i \pi^{m_i} - \delta_i \pi^{n_i} \geq 0, \forall n_i > m_i > 0$ and $\gamma_i \geq \delta_i \geq 0$. So in our case $k_i > 0, \forall i = 1, ..., 4$ and thus, $A > 0$. From this we conclude that $V(\pi) - V^D(\pi) > 0, \forall \pi \in]0;1]$.

ii) Simply replace $\pi = 0$ in the expression $V(\pi) - V^D(\pi)$ and find that $V(0) - V^D(0) = 0$ ■

Proof of proposition 4. First we prove that

$$x_{PC}^{1^*} = \frac{V_{S_{PC}} - V_{FF_{PC}} + \pi}{1 + V_{S_{PC}} - V_{SS_{PC}} - V_{FF_{PC}} + \pi} \in [0;1]$$

i) $x_{PC}^{1^*} \geq 0$

$$\Rightarrow V_{S_{PC}} - V_{FF_{PC}} + \pi \geq 0$$

$$\frac{\pi_2^2}{2} + \pi - \frac{\pi^2}{2\left(1+\pi\right)^2} + \pi \geq 0$$

$$\frac{\pi_2^2}{2} + \pi + \frac{2\pi + 3\pi^2 + 2\pi^3}{2\left(1+\pi\right)^2} \geq 0$$

$$\Rightarrow x_{PC}^{1^*} \geq 0, \forall \pi \geq 0 \text{ and } \pi_2 \in [0;1]$$

ii) $x_{PC}^{1^*} \leq 1$

$$\Rightarrow \frac{V_{S_{PC}} - V_{FF_{PC}} + \pi}{1 + V_{S_{PC}} - V_{SS_{PC}} - V_{FF_{PC}} + \pi} \leq 1$$

$$\Leftrightarrow V_{S_{PC}} - V_{FF_{PC}} + \pi \leq 1 + V_{S_{PC}} - V_{SS_{PC}} - V_{FF_{PC}} + \pi$$

$$1 - V_{SS_{PC}} \geq 0$$

$$\frac{2 + 4\pi_2 + \pi_2^2}{2\left(1+\pi_2\right)^2} \geq 0$$

$$\Rightarrow x_{PC}^{1^*} \leq 1, \forall \pi \geq 0 \text{ and } \pi_2 \in [0;1]$$

Finally we proved that $x_{PC}^{1^*} \in [0;1], \forall \pi \geq 0$ and $\pi_2 \in [0;1]$.

Second we show that $x_{PC}^{1^*}$ is a non decreasing function in both π and π_2.

i) $\frac{\partial x_{PC}^{1^*}}{\partial \pi} \geq 0$

$$\frac{\partial x_{PC}^{1^*}}{\partial \pi} = \frac{\left(\frac{\partial V_{S_{PC}}}{\partial \pi} - \frac{\partial V_{FF_{PC}}}{\partial \pi} + 1\right)\left(1 - V_{SS_{PC}}\right)}{\left(1 + V_{S_{PC}} - V_{SS_{PC}} - V_{FF_{PC}} + \pi\right)^2}$$

As we know that $1 - V_{SS_{PC}}$ is positive, we just have to show that $\frac{\partial V_{S_{PC}}}{\partial \pi} - \frac{\partial V_{FF_{PC}}}{\partial \pi} + 1 \geq 0$

$$\frac{\partial V_{S_{PC}}}{\partial \pi} - \frac{\partial V_{FF_{PC}}}{\partial \pi} + 1 = 2 - \frac{\pi}{\left(1+\pi\right)^3} \geq 0$$

$$\Rightarrow \frac{\partial x_{PC}^{1^*}}{\partial \pi} \geq 0, \forall \pi \geq 0 \text{ and } \pi_2 \in [0;1]$$

ii) $\frac{\partial x_{PC}^{1^*}}{\partial \pi_2} \geq 0$

$$\frac{\partial x_{PC}^{1^*}}{\partial \pi_2} = \frac{\frac{\partial V_{S_{PC}}}{\partial \pi_2}\left(1 - V_{SS_{PC}}\right) + \frac{\partial V_{SS_{PC}}}{\partial \pi_2}\left(V_{S_{PC}} - V_{FF_{PC}} + \pi\right)}{\left(1 + V_{S_{PC}} - V_{SS_{PC}} - V_{FF_{PC}} + \pi\right)^2}$$

Both terms in the numerator are positive for any $\pi \geq 0$ and $\pi_2 \in [0;1]$. Thus,

$$\frac{\partial x_{PC}^{1^*}}{\partial \pi_2} \geq 0, \forall \pi \geq 0 \text{ and } \pi_2 \in [0;1]$$

■

Proof of proposition 6. We denote the difference of the value functions $V_{PC}(\pi, \pi_2) - V_{PC}^D(\pi, \pi_2)$ by $\Delta V_{PC}(\pi, \pi_2)$. First of all we must prove that this function is continuous at any point for $\pi \geq 0$ and $\pi_2 \in [0;1]$. $\Delta V_{PC}(\pi, \pi_2)$ can be expressed as a fraction of two polynomials $f(\pi, \pi_2)$ and $g(\pi, \pi_2)$ such that

$$\Delta V_{PC}(\pi, \pi_2) = \frac{f(\pi, \pi_2)}{g(\pi, \pi_2)}$$

If we compute the explicit expression of $\Delta V_{PC}(\pi, \pi_2)$, we can see that both $f(\pi, \pi_2)$ and $g(\pi, \pi_2)$ are closed form functions for $\pi \geq 0$ and $\pi_2 \in [0;1]$ and thus are both continuous at any point for $\pi \geq 0$ and $\pi_2 \in [0;1]$. Since $\Delta V_{PC}(\pi, \pi_2)$ is a fraction of two closed form functions, $\Delta V_{PC}(\pi, \pi_2)$ is also a closed form function for $\pi \geq 0$ and $\pi_2 \in [0;1]$. From this we conclude that $\Delta V_{PC}(\pi, \pi_2)$ is also continuous at any point for $\pi \geq 0$ and $\pi_2 \in [0;1]$.

i) In order to prove the first point of proposition 6, we can show that

$$\lim_{\pi \to \infty} \Delta V_{PC}(\pi, \pi_2) = \frac{-\pi_2^2}{2\left(1 + \pi_2\right)^2} \leq 0, \forall \pi_2 \in [0;1]$$

and

$$\frac{\partial \lim\limits_{\pi \to \infty} \Delta V_{PC}(\pi, \pi_2)}{\partial \pi_2} = \frac{-\pi_2}{\left(1 + \pi_2\right)^3} \leq 0, \forall \pi_2 \in [0;1]$$

Thus

$$V_{PC}^D(\pi \to \infty, \pi_2) \geq V_{PC}(\pi \to \infty, \pi_2), \forall \pi_2 \in [0;1]$$

ii) As $\Delta V_{PC}(\pi, \pi_2)$ is continuous at any feasible values for π and π_2, $\lim\limits_{\pi_2 \to 1} \Delta V_{PC}(\pi, \pi_2) = \Delta V_{PC}(\pi, \pi_2 = 1) = \Delta V_{PC}(\pi)$. The explicit expression

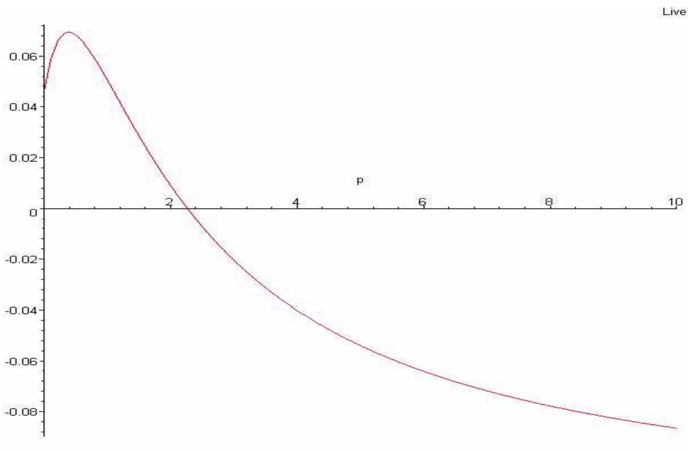

Figure 10: plot of $\Delta V_{PC}\left(\pi\right)$

of $\Delta V_{PC}\left(\pi\right)$ is:

$$\begin{aligned}\Delta V_{PC}\left(\pi\right) &= \frac{-0.125(\pi-2.2658)(\pi+1.8007)(\pi+1)^2(\pi+.7173)}{(\pi+.5232)^2(\pi+.4780)^2}* \\ &\quad *\frac{(\pi+.5002)(\pi+.2781)(\pi+.2229)}{(\pi^2+2.1018\pi+2.1504)^2}* \\ &\quad *\frac{(\pi^2+1.8644\pi+1.3577)(\pi^2+1.7572\pi+1.7888)}{(\pi^2+1.9595\pi+1.4384)^2}\end{aligned}$$

As the denominator of this function is always positive $\forall\pi > 0$, $\Delta V_{PC}\left(\pi\right)$ is negative or equal to zero if and only if $\pi \geq \pi_0 = 2.2658$. Figure 8 shows a plot of $\Delta V_{PC}\left(\pi\right)$.

Thus if both π and π_2 are both sufficiently large then $V_{PC}\left(\pi,\pi_2\right) - V_{PC}^{D}\left(\pi,\pi_2\right) \leq 0$. ■

Proof of proposition 7. As $\Delta V_{PC}\left(\pi,\pi_2\right)$ is continuous,

$$\lim_{\pi\to 0}\Delta V_{PC}\left(\pi,\pi_2\right) = \Delta V_{PC}\left(\pi=0,\pi_2\right) = \Delta V_{PC}\left(\pi_2\right)$$

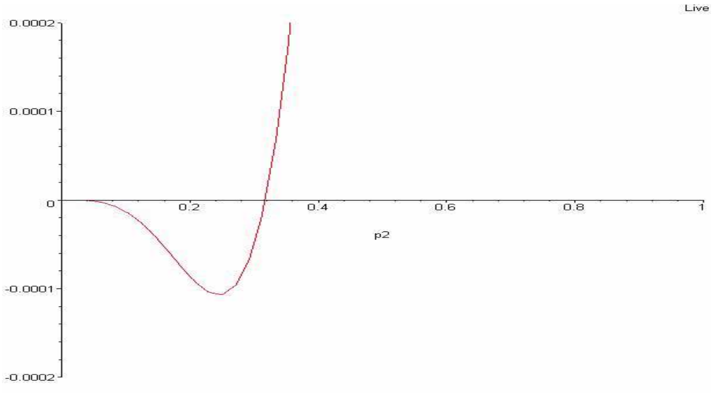

Figure 11: plot of $\Delta V_{PC}\left(\pi_2\right)$

The true expression for $\Delta V_{PC}\left(\pi_2\right)$ is:

$$\begin{aligned}\Delta V_{PC}\left(\pi_2\right) &= \frac{0.2770\pi_2^4(\pi_2-.3175)(\pi_2+4.4208)(\pi_2+2.0672)}{(\pi_2+1.7836)^2(1+\pi_2)^2(\pi_2+.6060845385)^2} * \\ &\quad * \frac{(\pi_2+1.6824)(\pi_2+.6202)(\pi_2+.5882)}{(\pi_2^2+1.3333\pi_2+.6667)^2} * \\ &\quad * \frac{(\pi_2^2+1.4999\pi_2+.7498)(\pi_2^2+.6387\pi_2+1.1983)}{(\pi_2^2-.3897\pi_2+1.85015)^2}\end{aligned}$$

This equation admits two solutions such that $\pi_2 \in [0;1]$: $\pi_2 = \{0; 0.3175\}$. The first solution involves an equality of the two value functions since both are equal to zero if $\pi_2 = \pi = 0$. The second solution, which we denote by $\pi_{2_0} = 0.3175$ is more interesting. As all the other factors as well as the denominator of the function above are positive $\forall \pi_2 \in [0;1]$, we can focus on the remaining factor $(\pi_2 - .3175) = (\pi_2 - \pi_{2_0})$. Thus if π_2 is sufficiently small, that is $\pi_2 \leq \pi_{2_0}$, then $\Delta V_{PC}\left(\pi_2\right) \leq 0$ and the firms share their knowledge. Figure 9 shows the shape of $\Delta V_{PC}\left(\pi_2\right)$. ■

Proof of proposition 8. $x_{PC}^{1*} \geq x_{PC}^{1^{D}*}$

$$\frac{V_{S_{PC}} - V_{FF_{PC}} + \pi}{1 + V_{S_{PC}} - V_{SS_{PC}} - V_{FF_{PC}} + \pi} \geq \frac{V_{SS_{PC}} - V_{FF_{PC}} + \pi}{1 + V_{SS_{PC}} - V_{FF_{PC}} + \pi}$$

After a few maths, we reduce this expression, which gives us

$$V_{SS_{PC}}^2 + V_{SS_{PC}}\left(\pi - V_{FF_{PC}}\right) + \left(V_{S_{PC}} - V_{SS_{PC}}\right) \geq 0$$

$$V_{SS_{PC}}^2 + V_{SS_{PC}}\left(\pi - \frac{\pi^2}{2\left(1+\pi\right)^2}\right) + \frac{\pi_2^2}{2}\left(1 - \frac{1}{\left(1+\pi_2\right)^2}\right) \geq 0$$

Obviously, this is true for any $\pi \geq 0$ and $\pi_2 \in [0;1]$. Thus, the investment strategy in the first stage is higher under competitive behavior than under disclosure behavior. $\blacksquare$

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