

Puzzling Comovements between Output and Interest Rates? Multiple Shocks are the Answer.*

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Abstract

Stylized facts on output and interest rates in the U.S. have so far proved hard to match with business cycle models. But these findings do not acknowledge that the economy might well be driven by different shocks, and by each in different ways. I condition covariances of output, nominal and real interest rate on two types of shocks: Technology and monetary policy.

The *conditional* results square with standard models. The “puzzling” overall comovements come from the interaction of various shocks. This calls more for looking at the composition of shocks and less for tweaking transmission mechanisms.

In particular, I find that technology and monetary shocks influence the correlation between output and interest rates in opposing ways. Conditional on technology, the real rate is pro-cyclical and a positive leading indicator. Conditional on monetary shocks it is counter-cyclical and a negative leading indicator. Non-technology and non-monetary shocks play an important role for movements in the nominal rate.

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1 Introduction

The relationship between output and interest rates has long been important to macroeconomists and policymakers alike. But basic stylized facts on their comovements in U.S. data have proved difficult to match within a variety of modern business cycle models. For instance, King and Watson (1996) study three models: a real business cycle model, a sticky price model, and a portfolio adjustment cost model. They report that this battery of modern dynamic models fails to match the business cycle comovements of real and nominal interest rates with output:

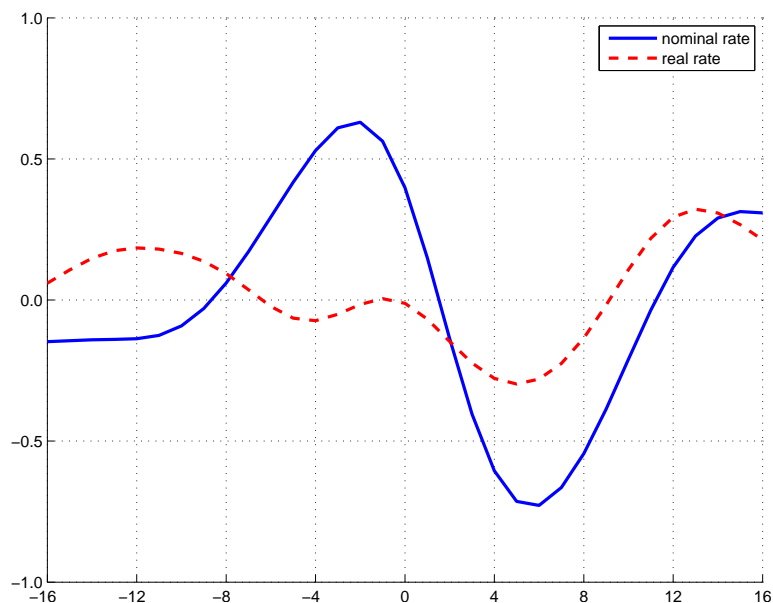
While the models have diverse successes and failures, none can account for the fact that real and nominal interest rates are “inverted leading indicators” of real economic activity.¹

Calling interest rates inverted leading indicators refers to their negative correlation with future output. These correlations are typically measured once the series have been passed through a business cycle filter.² Amongst the diverse failures mentioned by King and Watson, RBC models generate mostly a pro-cyclical real rate. But in the data, the real rate is clearly not pro-cyclical, it has a zero (or even negative) contemporaneous covariance with output. As mentioned already, it is also a negative leading indicator. This commonly found pattern of correlation between bc-filtered output and short-term interest rates is depicted

¹King and Watson (1996, p.35). The inverted leading indicator property has been the subject of various empirical studies, for example Sims (1992) and Bernanke and Blinder (1992). The expression “negative leading indicator” is synonymous.

²When it can be applied without confusion, I use the phrase “business cycle filter”, or short “bc-filter”, to describe the bandpass filters developed and applied in Baxter and King (1999) and Stock and Watson (1998) or the filter of Hodrick and Prescott (1997, “HP”) since each eliminates nonstationary and other low frequency components from a time series. These filters differ mainly in that the typical bandpass filters eliminates not only cycles longer than 32 quarters but also those shorter than 6 quarters, while this latter high-frequency component is retained in the HP filter.

Figure 1: Lead-lag Correlations for Output and Interest Rates



Note: $\text{cor}(\tilde{y}_t, \tilde{x}_{t-k})$ where \tilde{y}_t is bandpass-filtered per-capita output and \tilde{x}_t is bandpass-filtered nominal, respectively real rate. This ex-ante real rate is constructed from the VAR described in Section 2 as $r_t = i_t - E_t\pi_{t+1}$. Lags in quarters. U.S. data 1966–1996.

in Figure 1.³

What is the correct conclusion from a mismatch between implications from a dynamic model and stylized facts? Modern dynamic models always involve a joint specification of fundamental economic structure and driving processes. Model outcomes, such as the output-interest rate correlation, involve the compound effect of these two features. Yet, when “puzzling” findings are taken as evidence against a particular structural feature – such as sticky prices or portfolio

³This evidence is broadly in line with previous studies, see for instance the stylized facts collected by Stock and Watson (1998, Table 2) for bandpass-filtered U.S. data. Stock and Watson even report a strongly negative correlation. The difference with my estimates is due to a different choice of price deflator in the real rate construction. I use the more modern PCE deflator instead of the CPI as they do. Dotsey, Lantz, and Scholl (2003) discuss this sensitivity. I document the robustness of my *conditional* results to the choice of inflation measure in the appendix, which is available on my website.

adjustment costs – it is typically not acknowledged that the economy might alternatively be driven by different types of shocks that yield different effects within the given structure. Yet, more carefully, it is simply unclear whether dynamic models fail (or succeed) because of their transmission mechanisms or because of the nature of their driving forces.

To shed more light on this important issue, I provide empirical evidence about output-interest rate comovement conditional on two types of structural shocks that also drive the models of King and Watson (1996): Technology and monetary shocks. Rotemberg (1996), Gali (1999) and den Haan (2000) stress the importance of looking at conditional comovements in the context of the comovement of output with either prices or hours. In applying this general idea to output and interest rates, my specific approach is motivated by the fact that the “puzzle” in this area is typically expressed in terms of bc-filtered data.

There are striking results of my decomposition, which are reported in section 3 using plots analogous to Figure 1:

- After conditioning on technology, the real rate is pro-cyclical and a positive leading indicator – just the opposite of its unconditional behavior. This is not only matched by standard RBC models as in King and Watson (1996), but also by the technology channel of textbook New-Keynesian models as laid out by Gali (2003), Walsh (2003) or Woodford (2003).
- Conditional on monetary shocks the real rate is counter-cyclical and a negative leading indicator, which squares with the simple New-Keynesian models, too.⁴

⁴Money is neutral in RBC models, so they have not much to say here. Conditional on monetary shocks, output remains in steady state and correlations are zero.

Thus, the “output-interest rate puzzle” is largely resolved by a combination of comovements from two widely-studied shocks. Opposing effects of shocks to “supply” and “demand” are a general theme in Keynesian models (Bénassy 1995). But, the approach also reveals that there is a large unexplained component, when it comes to nominal interest rates. This means that focusing only on shocks to technology and monetary policy will not be sufficient to model the behavior of output and interest rates, in particular for the inflation component in the nominal rate.

The backbone of my calculations is a VAR for the joint process of (unfiltered) output, nominal and real interest rate. The VAR serves both as a platform for identifying the structural shocks and to model the bc-filtered covariances and correlations. The identified shocks are shocks to the unfiltered data. For instance, the technology shock has a permanent effect on output but it might also have important effects on economic fluctuations. The point of bc-filtered statistics is to judge models solely on those cyclical properties, not on their implications for growth (Prescott 1986). In this vein, the VAR is used to trace out the effects of shocks to the bc-filtered components of output and interest rates. This is done analytically using a frequency domain representation for the VAR and the bc-filters.

To measure technology shocks, I identify innovations to the permanent component of per-capita output. If per-capita hours are trend-stationary, this is equivalent to the permanent component of labor-productivity, which is the customary definition of technology shocks (Gali 1999; Christiano, Eichenbaum, and Vigfusson 2003). Of course, unit roots are notoriously hard to distinguish in finite sample. In an appendix available on the web⁵, I report unit root tests favoring

⁵The web-appendix is available at <http://www.elmarmertens.org>.

the view of stationary hours in my sample. Otherwise, my results on “technology shocks” can still safely be read as pertaining to permanent shocks in output. Robustness to alternative specifications is documented in the appendix as well.

Monetary shocks are identified as innovations to the federal funds rate which are unexplainable from Fed’s intentions and anticipations about future inflation and activity. Romer and Romer (2004) compute a time series of this policy measure based on FOMC minutes and the Fed’s Greenbook, which I relate to my VAR’s forecast errors of the short-term nominal interest rate (T-Bill). Neatly, the Romer measure is practically uncorrelated to the technology shocks – just as it should! The two identification strategies barely interfere with each other. My technology shocks would be estimated to be practically the same when disregarding the Romer measure and vice versa.

The remainder of this paper is structured as follows: Section 2 lays out my VAR framework for identification of the shocks as well as for decomposing the filtered covariances. Results are presented in Section 3. Section 5 concludes the study.

2 Empirical Methodology

The variables of interest to my study are the logs of per-capita output⁶, the nominal as well as the real interest rate:

$$Y_t = \begin{bmatrix} y_t \\ i_t \\ r_t \end{bmatrix} \quad (1)$$

⁶All quantity variables shall be per-capita without further mention.

Let us call their bc-filtered component \tilde{Y}_t . The goal is to model and estimate how structural shocks induce comovements between the elements of \tilde{Y}_t .

The backbone of all my calculations is a VAR. Owing to the real interest rate, Y_t is not fully observable. So the VAR is not run directly over Y_t but rather over a vector of observables X_t . As a benchmark, I specify a simple three-variable system using output growth, inflation and the nominal rate:

$$X_t = \begin{bmatrix} \Delta y_t \\ \pi_t \\ i_t \end{bmatrix} \quad (2)$$

The dynamics of X_t are captured by a p -th order VAR:⁷

$$A(L)X_t = e_t = Q \varepsilon_t \quad (3)$$

where

$$A(L) = \sum_{k=0}^p A_k L^k, \quad A_0 = I$$

and $E_{t-1}\varepsilon_t = 0, \quad E_{t-1}\varepsilon_t\varepsilon_t' = I$

The coefficients A_k and forecast errors e_t can be estimated using OLS. Identification of the structural shocks ε_t will be concerned with pinning down Q . Since fewer shocks are identified than the VAR has equations, this pins down only the first two columns of Q . There remains an unidentified component, which can be

⁷For convenience, I dropped the constants such that X_t is mean zero. This is without loss of generality since estimating a VAR from demeaned data is equivalent to running a VAR with constants over the actual data.

arbitrarily orthogonalized,⁸ but without structural interpretation.

Y_t can be constructed from X_t by applying a linear filter. This filter computes the real rate according to the Fisher equation $r_t = i_t - E_t \pi_{t+1}$:

$$Y_t = H(L)X_t$$

where

$$H(L) = \begin{bmatrix} (1-L)^{-1}h_{\Delta y} \\ h_i \\ h_i - h_\pi \left(\sum_{k=1}^p A_k L^{k-1} \right) \end{bmatrix}$$

and $h_{\Delta y}$, h_i and h_π are selection vectors such that $\Delta y_t = h_{\Delta y} X_t$ and so on.

The remainder of this section describes the following: First, how the structural shocks are identified (Sections 2.1 and 2.2). This gives us Q and the conditional dynamics of the *unfiltered* variables can be computed from $Y_t = H(L)A(L)^{-1}Q \varepsilon_t$. Second, how to apply a bc-filter to the structural components of Y_t to obtain the decomposition of their auto-covariances (Section 2.3).

2.1 Technology Shocks

Following a now common notion introduced by Gali (1999), technology shocks are the only innovation to the permanent component of *labor-productivity* (output

⁸Formally, stack the identified elements of ε_t into $\tilde{\varepsilon}_t$ and call the unidentified component η_t :

$$e_t = \tilde{Q}\tilde{\varepsilon}_t + \eta_t$$

Since $E\eta_t\eta_t'$ is rank deficient it is only positive *semi*-definite and a Cholesky decomposition is not applicable (it requires strict positive definiteness). An algorithm which accounts for the singularity is for instance the singular value decomposition, which boils down to an eigenvector-eigenvalue decomposition in the case of square matrices. Key for the decomposition is that η_t is by construction orthogonal to the two identified shocks in $\tilde{\varepsilon}_t$.

per hour).⁹ A contentious matter in implementing this long-run restriction is whether hours are assumed to be stationary in levels or differences (Christiano, Eichenbaum, and Vigfusson 2003; Gali and Rabanal 2004). Hours themselves are not of concern for my study. But when per-capita hours, n_t , are stationary, the permanent component of labor-productivity equals the permanent component of (per-capita) output.

For the purpose of this paper, I identify technology shocks as driving the permanent component of *per-capita output*. This is backed up by several observations reported in the web-appendix: First, unit root and VECM tests favor the stationarity of hours in my sample. Second, the results are qualitatively robust, when a stochastic trend in hours is allowed¹⁰. Finally, the predictions of standard models – RBC or New Keynesian – for output and interest rates remain identical, even when the permanent shocks to output were not only accounted for by technology shocks but also by other candidates such as government spending or changes in the workforce composition (Francis and Ramey 2005).

The identifying restriction is that the first row of $A(1)^{-1}Q$ is full of zeros, except an entry of one for the (standardized) technology shock in its first column.

$$A(1)^{-1}Q = \begin{bmatrix} 1 & 0 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

Together with the orthogonality of the structural shocks, this identifies the first column of Q . It is computed using the method of Blanchard and Quah (1989)¹¹.

⁹See Fisher (2002) and Greenwood, Hercowitz, and Krusell (1997) for an alternative definition of technology as specifically improving investments.

¹⁰Albeit the quantitative significance of the results is reduced. In my view, this also reflects the increased estimation uncertainty due adding a new variable (hours) to the VAR system.

¹¹An alternative method, yielding the same results, would be the instrumental variables

The robustness of results to identifying technology shocks from labor productivity whilst allowing for a stochastic trend in hours is documented in the web-appendix.

2.2 Monetary Shocks

Again following standard conventions, monetary shocks are defined as unexpected deviations from endogenous policy. With the Fed Funds rate as policy instrument, they are unexpected Taylor-rule residuals, just as in Christiano, Eichenbaum, and Evans (1998) or Rotemberg and Woodford (1997). Strictly speaking, I do not identify such shocks myself. Rather, the measure of Romer and Romer (2004) is hooked up to my VAR. This allows to keep the VAR small, whilst the measure of the Romers takes care of the Fed’s information about future activity and inflation – variables which typically influence endogenous policy. Without that information, my small VAR would likely produce the “price puzzle” by confounding an anticipatory increase in interest rates with an exogenous policy move¹².

Romer and Romer have recently constructed a measure from FOMC minutes and the Fed’s Greenbook which explicitly accounts for the Fed’s policy intentions and for the Fed’s anticipation of future inflation and activity. The series has been constructed for each FOMC meeting from 1964 to 1997. It is based on a series of “Intended Fed Funds rates”¹³ for each FOMC meeting. Their policy measure is

regressions of Shapiro and Watson (1988). This framework is more amenable to include over-identifying restrictions, such as the orthogonality of technology and monetary shocks. See the web-appendix for a description.

¹²The VAR would spuriously document inflation to rise in response to a monetary tightening. That is the price puzzle. A partial but classic response would be to include inflation-forecasting variables, like commodity prices (Christiano, Eichenbaum, and Evans 1998). See Hanson (2004) and Giordani (2004) for a critical discussion.

¹³In constructing the series from FOMC minutes (prior to official targets) Romer and Romer (2004) found that even when the Funds rate was not the official policy instrument, policy makers’ thinking was fairly well shaped around informal fund rate targets. This supports the identification of policy shocks from interest rates over a period which featured different schemes of official monetary policy making (Bernanke and Blinder 1992; Bernanke and Mihov 1998).

the residual from a regression of these policy intentions on Greenbook forecasts of activity and inflation. The details are described by Romer and Romer (2004) and the insightful discussion by Cochrane (2004).

The first result emerges when their series is aggregated into quarterly data¹⁴: It is virtually uncorrelated to the technology shocks identified above – just as it should. The sample correlation is 0.08. The web-appendix lays out a test strategy following Shapiro and Watson (1988) and cannot reject that the Romer measure is orthogonal to the technology shocks. For convenience, I even impose orthogonality *in sample* by projecting the Romer measure off the technology shocks.

Since monetary policy is typically assumed to operate with a lag, the Romer measure is projected off the forecast error for inflation, too.¹⁵ Without this additional short-run restriction on inflation, the impulse responses would display a positive price response to a monetary tightening. (The results on filtered covariances are however unaffected.) Formally, the normalized monetary shock is the standardized residual ε_t^m in

$$\varepsilon_t^{\text{Romers}} = \beta' \begin{bmatrix} \varepsilon_t^z \\ e_t^\pi \end{bmatrix} + \sigma_m \varepsilon_t^m$$

where $\varepsilon_t^{\text{Romers}}$ is the measure of the Romers, ε_t^z the technology shock and e_t^π is the forecast error of inflation (second element of e_t in (3)). The second column of Q is then simply filled up with the slopes of regressing the forecast errors e_t

¹⁴The aggregation proceeds analogously to what Romer and Romer (2004) do in constructing their monthly data. All residuals pertaining to FOMC meetings within a quarter are added up to yield the quarterly measure.

¹⁵Romer and Romer (2004) also impose a lagged response of output and inflation to their policy measure.

onto the time series of monetary shocks ε_t^m .¹⁶

2.3 Decomposition of BC-Filtered Covariances

Summarizing the previous discussion, the impulse responses of the unfiltered variables in Y_t are given by $H(L)A(L)^{-1}Q$. These do not only trace out the business cycle responses of Y_t to the structural shocks ε_t , but also how the shocks induce growth as well as high-frequency noise. The motivation for bc-filtering is now to focus only on the business cycle effects.¹⁷ Formally, it remains to apply a bc-filter and to decompose the filtered lead-lag covariances into the contributions of the structural shocks. The computations are straightforward to perform in the frequency domain. A classic reference for the necessary tools is Priestley (1981). Altig et al. (2004) employ similar techniques.

The analysis is applicable to a wide class of bc-filters, including the HP-Filter, the approximate bandpass filter of Baxter and King (1999) as well as the exact bandpass filter. My empirical results are not sensitive to a particular choice amongst these filters. For the computations it is key that the bc-filter can be written as a linear, two-sided, infinite horizon moving average whose coefficients

¹⁶This regression follows from $e_t = Q\varepsilon_t$ and the orthogonality of the structural shocks. Even though the regression of e_t on ε_t is multivariate, it follows from the Frisch-Waugh-Lowell Theorem, that the regression slopes can be computed from separate, univariate regressions as well.

¹⁷Business cycle filters have also been criticized for creating spurious cycles, originally by Harvey and Jaeger (1993) and followed by Cogley and Nason (1995) as well as in the discussion between Canova (1998a, 1998b) and Burnside (1998). Whilst most of these papers focused on the HP filter, their analysis also applies to the bandpass filter. But the bc-filtered statistics employed here can be perfectly justified from the perspective of model evaluation in the frequency domain: The goal is not to match data and model over all spectral frequencies, but only over a subset which is associated with “business cycles”. For the U.S. this is typically taken to be 6 to 32 quarters following the NBER definitions of Burns and Mitchell (Baxter and King 1999; Stock and Watson 1998). Formal concepts of model evaluation in this vein have been advanced by Watson (1993), Diebold, Ohanian, and Berkowitz (1998), as well as Christiano and Vigfusson (2003). Using the concept of the pseudo-spectrum this extends also to nonstationary variables, notwithstanding the analysis of Harvey and Jaeger.

sum to zero:¹⁸

$$\begin{aligned}\tilde{Y}_t &\equiv B(L)Y_t \\ \text{where } B(L) &= \sum_{k=-\infty}^{\infty} B_k L^k \\ \text{and } B(1) &= 0\end{aligned}$$

The bandpass-filter is a such a symmetric moving average. It is explicitly defined in the frequency domain and most of my calculations are carried out in the frequency domain. For frequencies $\omega \in [-\pi, \pi]$, evaluate the filter at the complex number $e^{-i\omega}$ instead of the lag operator L . This is also known as the Fourier transform of the filter which represents it as a series of complex numbers (one for each frequency ω). Requiring $B(1) = 0$ sets the zero-frequency component of the filtered time series to zero. For instance, the bandpass filter¹⁹ passes only cycles between 6 and 32 quarterly observations:

$$B(e^{-i\omega}) = \begin{cases} 1 & \forall |\omega| \in \left[\frac{2\pi}{32}, \frac{2\pi}{6}\right] \\ 0 & \text{otherwise} \end{cases}$$

Since the bc-filtered variables in \tilde{Y}_t are covariance-stationary, their lead-lag covariances exist and so does their spectrum. They can be computed from the

¹⁸Of course, some coefficients B_k can be zero. So $B(L)$ could also be the first-difference filter. But meaningful bc-filters should also be symmetric, such that they have a zero phase shift. Otherwise, comovements over one frequency band, say business cycles, could be attributed by the filter to other frequencies, like growth.

¹⁹Alternatively, the HP filter approximates a high-pass filter (King and Rebelo 1993) and its Fourier transform is

$$B(e^{-i\omega}) = \frac{4\lambda(1 - \cos(\omega))^2}{1 + 4\lambda(1 - \cos(\omega))^2}$$

where λ is a smoothing parameter, conventionally set to 1600 for quarterly data. Likewise, the approximate Bandpass-Filter can be implemented by computing the fourier transform of the (truncated) lag polynomial $B(L)$ described by Baxter and King (1999).

VAR parameters and the filters $H(L)$ and $B(L)$. To ease notation, the impulse responses of Y are written as

$$C(L) \equiv H(L) A(L)^{-1} Q$$

respectively, after applying the bc-filter

$$\tilde{C}(L) \equiv B(L)C(L)$$

so that the bc-filtered spectrum can be expressed as

$$S_{\tilde{Y}}(\omega) = \tilde{C}(e^{-i\omega}) \tilde{C}(e^{-i\omega})' \quad (4)$$

For each frequency ω , this is simply a product of complex-numbered matrices²⁰. The lead-lag covariance matrices of \tilde{Y}_t can be recovered from the spectrum in what is known as an inverse Fourier transformation

$$\begin{aligned} \Gamma_{\tilde{Y}}^k &\equiv E\tilde{Y}_t\tilde{Y}_{t-k}' \\ &= \frac{1}{2\pi} \int_{\omega=-\pi}^{\pi} S_{\tilde{Y}}(\omega) e^{i\omega k} d\omega \end{aligned} \quad (5)$$

which can be accurately and efficiently computed using well-established algorithms²¹.

²⁰The transposes are conjugate transpose, i.e. they flip also the sign of the imaginary components.

²¹In Matlab for instance, fast Fourier algorithms are encoded in `fft` and `ifft`. For ω I use an evenly spaced grid over the unit circle with 1024 respectively 512 elements depending on the persistence of the VAR (the discrete fast Fourier algorithms behind `fft` and `ifft` work best for powers of 2). Here is a rule of thumb for the accuracy of the discretized Fourier transformation: Call n the number of grid points and λ the largest eigenvalue (in absolute terms) of the VAR's companion matrix. λ^n should be numerically close to zero to ensure accuracy over the entire range of frequencies. The reason is that discretizing the frequencies over $[-\pi; \pi]$ is analogous to approximating the complete dynamics by a finite number of impulse

Since the structural shocks are orthogonal to each other, the decomposition of the covariances $\Gamma_{\tilde{Y}}^k$ is straightforward. First, the spectrum is computed conditional on each shock. Then, the conditional lead-lag covariances follow from an inverse Fourier transformation, analogously to equation (5). To fix notation, the shocks are indexed by s and J_s is a square matrix, full of zeros except for a unit entry in its s 'th diagonal element. The spectrum conditioned on shock s is

$$S_{\tilde{Y}|s}(\omega) = \tilde{C}(e^{-i\omega}) J_s \tilde{C}(e^{-i\omega})' \quad (6)$$

Since $\sum_s J_s = I$ the conditional spectra add up to $S_{\tilde{Y}}(\omega)$.²² This carries over to the coefficients $\Gamma_{\tilde{Y}|s}^k \equiv E(\tilde{Y}_t \tilde{Y}_{t-k} | s)$ from the inverse Fourier transformation of $S_{\tilde{Y}|s}(\omega)$.

$$\sum_s \Gamma_{\tilde{Y}|s}^k = \Gamma_{\tilde{Y}}^k$$

This VAR framework is also capable of handling unit roots in Y_t . By construction, $H(L)$ and thus Y_t has a unit root such that $H(1)$ is infinite. For computing the bc-filtered spectrum $S_{\tilde{Y}}(\omega)$ in (4), $B(1) = 0$ takes precedence over this unit root. It is straightforward to check that $H(e^{i\omega})$ is well defined everywhere else, just except for the zero frequency. So we can think of the nonstationary vector Y_t having a pseudo-spectrum $S_Y(\omega) = C(e^{-i\omega}) C(e^{-i\omega})'$, which exists for every frequency on the unit circle except zero. Similar remarks apply to potential unit

responses. For stationary variables, the impulse responses ultimately converge to zero. The rule of thumb picks n large enough to capture this. Computations can be sped up dramatically using $S_{\tilde{Y}}(\omega) = S_{\tilde{Y}}(\omega)' = S_{\tilde{Y}}(-\omega)^T$ (where T is the simple, non-conjugate transpose) and by computing the spectrum only for frequencies where $B(e^{-i\omega}) \neq 0$.

²²Since I identify only two structural shocks but estimate a VAR with more than two variables, only the first two elements of ε_t have a structural interpretation whilst the others have been arbitrarily orthogonalized (see Footnote 8.). The unidentified components can be lumped together in a group using $J_{\text{Rest}} = \sum_{s \notin \{1,2\}} J_s$ instead of J_s in the formula above.

roots in the VAR such that some element(s) of $A(1)$ would be zero. As long as the solutions to the characteristic equation $A(z) = 0$ are on or outside, but not inside the unit circle, the computations above run through. Higher orders of integration, i.e. powers of unit roots $(1 - L)^d$ where d is an integer, thus fit in this framework as well.²³ Moreover, the web-appendix documents that most unit root tests favour a stationary specification of inflation and the nominal interest rate.

3 Empirical Results

This section presents the results for the three-variable benchmark VAR, equation (2) described in the previous section. Quarterly data is taken from FRED²⁴ on per-capita output (real GDP), inflation of the personal consumption expenditure index deflator (PCED)²⁵ and the average nominal yield on three-month T-Bills for the U.S. from 1964 to 1996. This sample is determined by the availability of the Romer shocks. Except for the interest rates all data is seasonally adjusted in this study. The VAR is estimated with 8 lags to ensure uncorrelated residuals. A detailed discussion of lag-length selection can be found in the web-appendix. After accounting for initial values, the sample covers the period from 1966 to 1996. My results are qualitatively the same for the HP or the bandpass filter. Here, results will be presented for the bandpass filter.

²³Even for a nonstationary VAR, the OLS estimates of its coefficients in $A(L)$ are consistent, they would even be super-consistent. The roots of $A(z)$ are the inverse of the eigenvalues to the VAR's companion form matrix. The only computational issue is that this cannot handle VARs whose *point estimates* imply companion eigenvalues outside the unit circle.

²⁴Federal Reserve Economic Data, maintained at <http://research.stlouisfed.org/fred2/> by the Federal Reserve Bank of St. Louis.

²⁵The web-appendix contains also results for other inflation measures: the CPI and the GDP deflator.

3.1 Real Interest Rate and Output

The key result is that technology shocks induce a strongly pro-cyclical real rate which is also a positive leading indicator for up to 4 quarters. This is depicted in Figure 2 which decomposes the filtered covariances between output and the real rate.

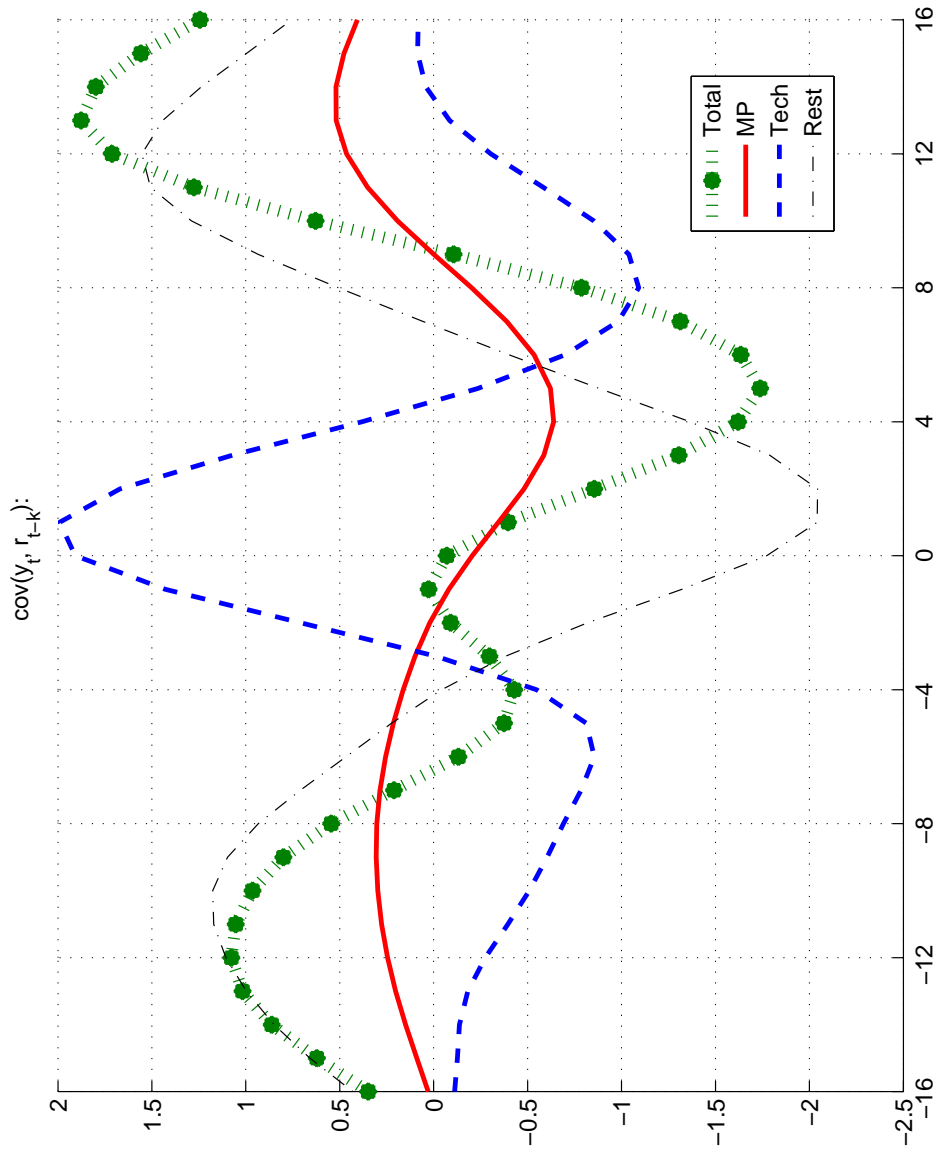
Figure 2 shows further that monetary shocks induce negative covariances at leads between zero and 8. Overall, monetary shocks appear to play a much smaller role in terms of explaining the overall covariation. Monetary shocks are essentially defined as the Fed rolling dice, so it is no wonder that their impact is comparably small.

Covariances add linearly, so they are a natural measure for the decomposition. But since they can be negative as well as positive, it is ambiguous to put a number like “percentage explained” on the decomposition. A case in point is the contemporaneous covariance between output and the real rate, which is unconditionally close to zero. It is the sum of a strong positive covariance induced by technology and an almost equally large negative contribution from the unidentified shocks (the monetary component is small). Clearly, the technology component appears large and important. But then it is also mirrored by an unexplained component of similar size.

Bootstrapped standard error bands²⁶ for the individual components are displayed in Figure 3 for technology shocks and Figure 4 for monetary shocks. Conditional on technology shocks, the real rate is not only pro-cyclical, but also significantly so. Likewise, monetary shocks significantly induce real rates to be inverted leading indicators of cyclical output. Figures 3 and 4 also plot the de-

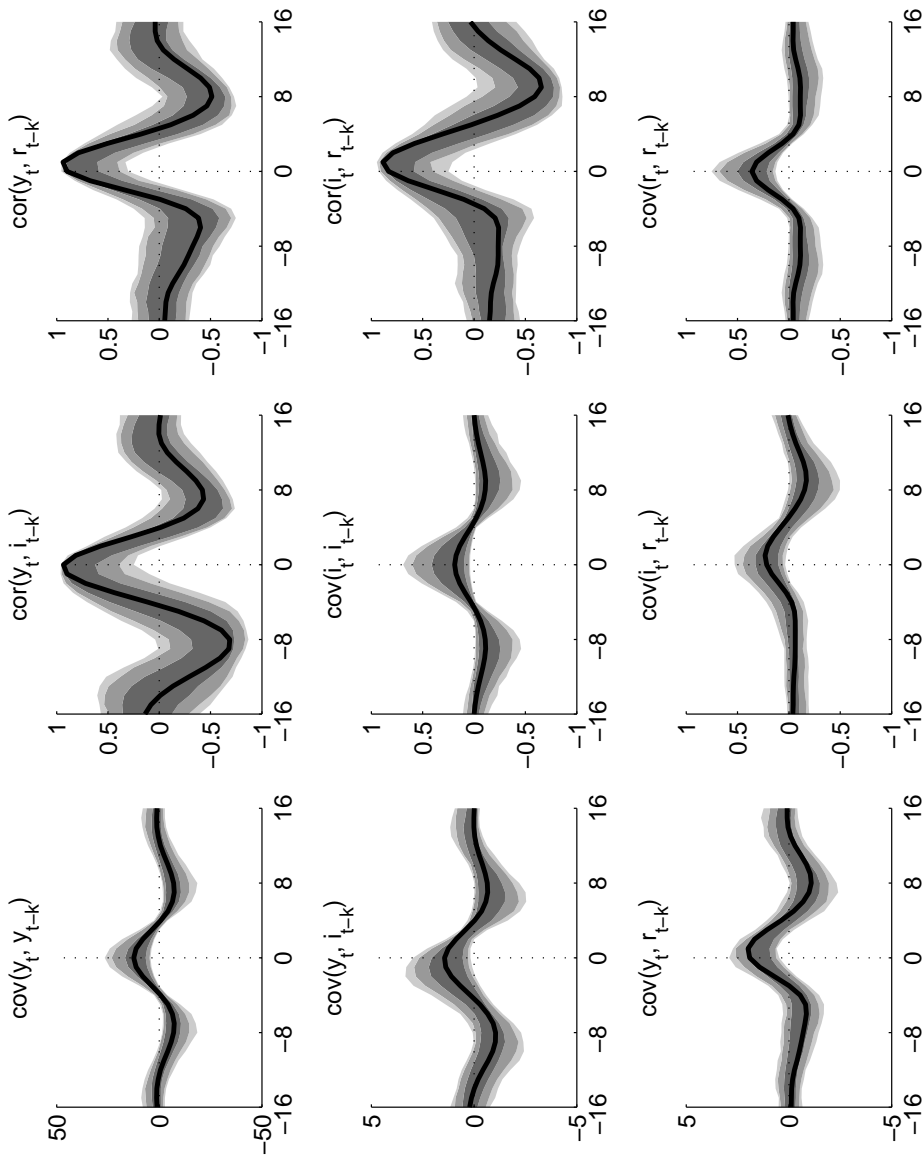
²⁶They are computed using 500 draws from the VAR process implied by the reduced-form point estimates. These are what Sims and Zha (1999) call “other-percentile bootstraps” or “flat-prior Bayesian posterior[s]” in the context of impulse response functions.

Figure 2: Covariance Decomposition for Real Rate and Output



Note: Bandpass filtered moments computed from benchmark VAR described in Section 2. Lags in quarters.

Figure 3: BC-Moments conditional on Technology Shock



Note: Covariances conditional on technology shocks for benchmark VAR described in Section 2. Correlations on the upper diagonal. Bandpass-filtered moments. Bootstrapped standard-errors bands (500 draws). Percentiles are shaded: 95% (light), 90% (middle) and 60% (dark). Lags in quarters.

composed *correlations* on their upper-diagonal panels. As throughout the paper, these correlations are between bandpass-filtered variables. Conditional on technology, the almost perfect correlation between real rate and current output is highly significant. In addition, that real rate is a significant, *positive* leading indicator for output about a year ahead. Conditional on monetary shocks, the real rate is a significant *inverted* indicator for output with a lead of three to seven quarters. (Their contemporaneous correlation is small and insignificant.)

These results describe the cyclical behavior between output and the real rate in terms of lead-lag covariances of after bc-filtering. They are corroborated by the impulse responses of the *unfiltered* variables. The right column of Figure 5 plots the impulse response of Y_t to a technology shock. By construction, it raises output permanently and significantly. In line with the growth in output, the real rate is increased for about two years before it returns to its steady state. In response to a monetary shock, output displays the typical hump-shaped contraction starting with a lag and lasting for about two to three years.²⁷ The real rate increases immediately and stays significantly above steady state for about two years. Because of the non-trivial effects of bc-filtering²⁸ it is not a foregone conclusion, that the picture emerging from the impulse responses should mirror the results for the bc-filtered comovements as it does here.

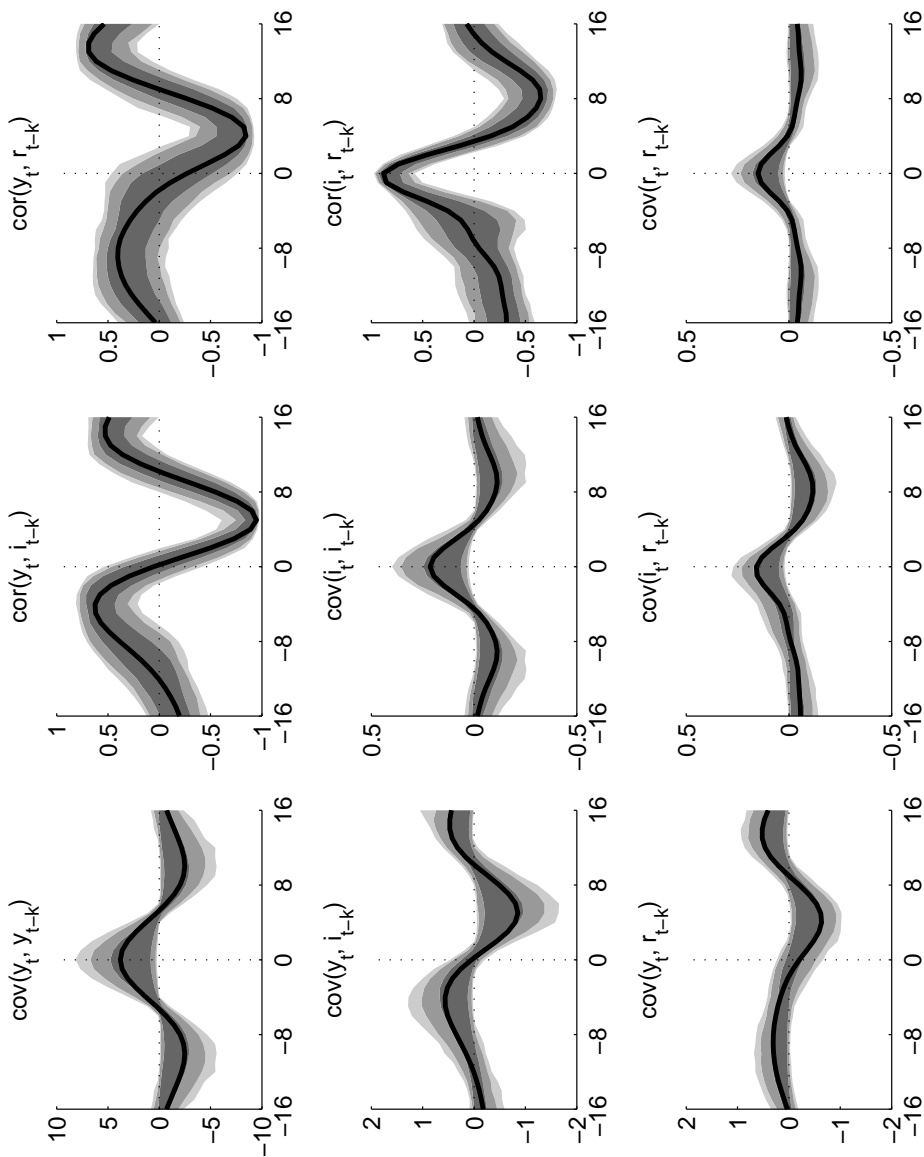
3.2 Nominal Interest Rate and Output

Turning to the nominal rate, the identified components play a much smaller, almost negligible role. This is shown in Figure 6, which is the analogue to Figure 2

²⁷This response, displayed in the top-left panel of Figure 5 for my quarterly data, is similar to what Romer and Romer (2004, Figure 2) estimate for monthly data using the same policy measure. Christiano, Eichenbaum, and Evans (1998, Figure 2) find similar responses for policy rules based on either interest rates or monetary aggregates.

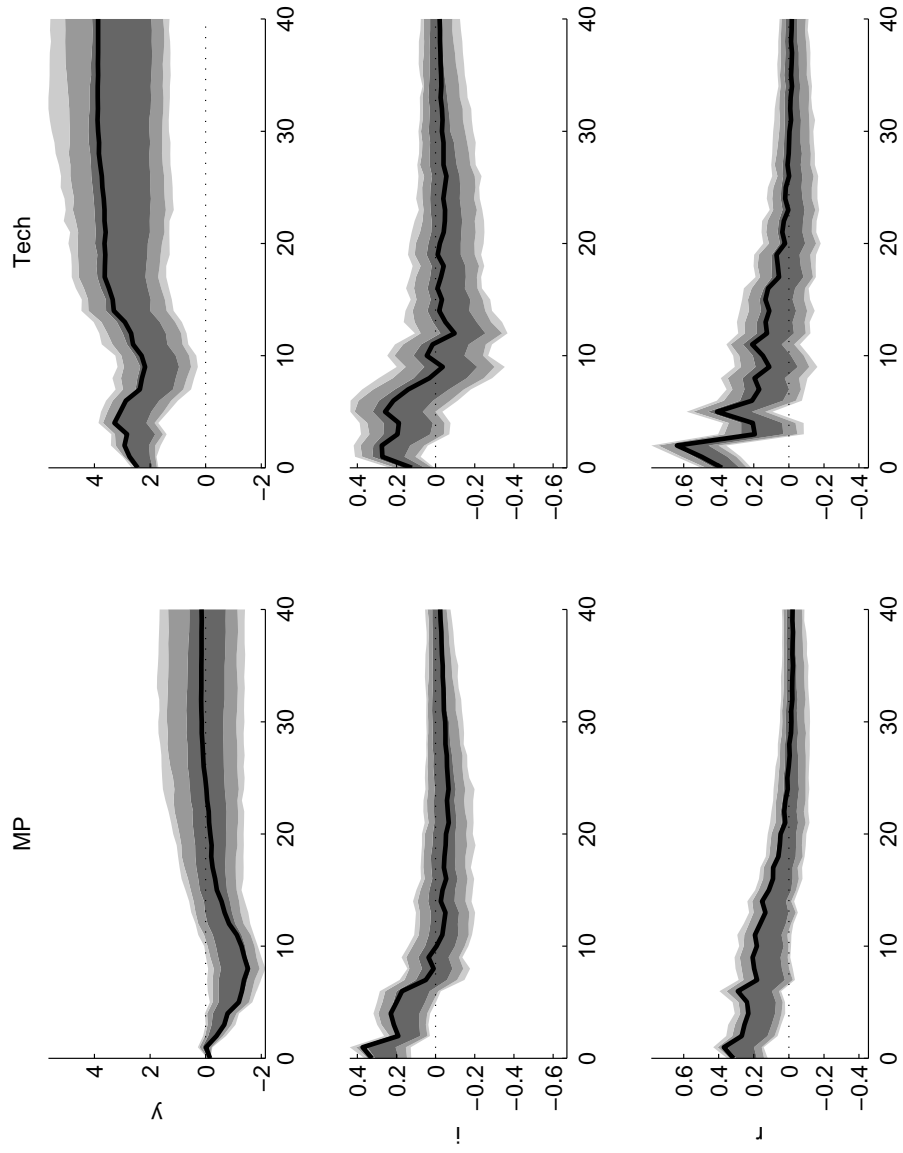
²⁸For a critical discussion see for instance Canova (1998a) or King and Rebelo (1993).

Figure 4: BC-Moments conditional on Monetary Shock



Note: Covariances conditional on monetary shocks for benchmark VAR described in Section 2. Correlations on the upper diagonal. Bandpass-filtered moments. Bootstrapped standard-errors bands (500 draws). Percentiles are shaded: 95% (light), 90% (middle) and 60% (dark). Lags in quarters.

Figure 5: Impulse Response Functions



Note: Estimates from benchmark VAR, equation (2) described in Section 2. Lags in quarters. Bootstrapped standard-errors bands (500 draws). Percentiles are shaded: 95% (light), 90% (middle) and 60% (dark).

for the real rate. The culprit is the low explanatory power of the identified shocks for inflation's cyclical. As witnessed by Figure 1, variations in expected inflation account for a large part of nominal rate variations. This makes the nominal rate inherit the low explanatory power.

As mentioned above, shocks to monetary policy are about unsystematic policy and plausibly small. What is striking is that technology shocks seem not to matter either. But if the Fed responds optimally to technology shocks, i.e. without creating inflation, this outcome is just to be expected. At least for the second half of my sample Gali, Lopez-Salido, and Valles (2003) find evidence for this.²⁹

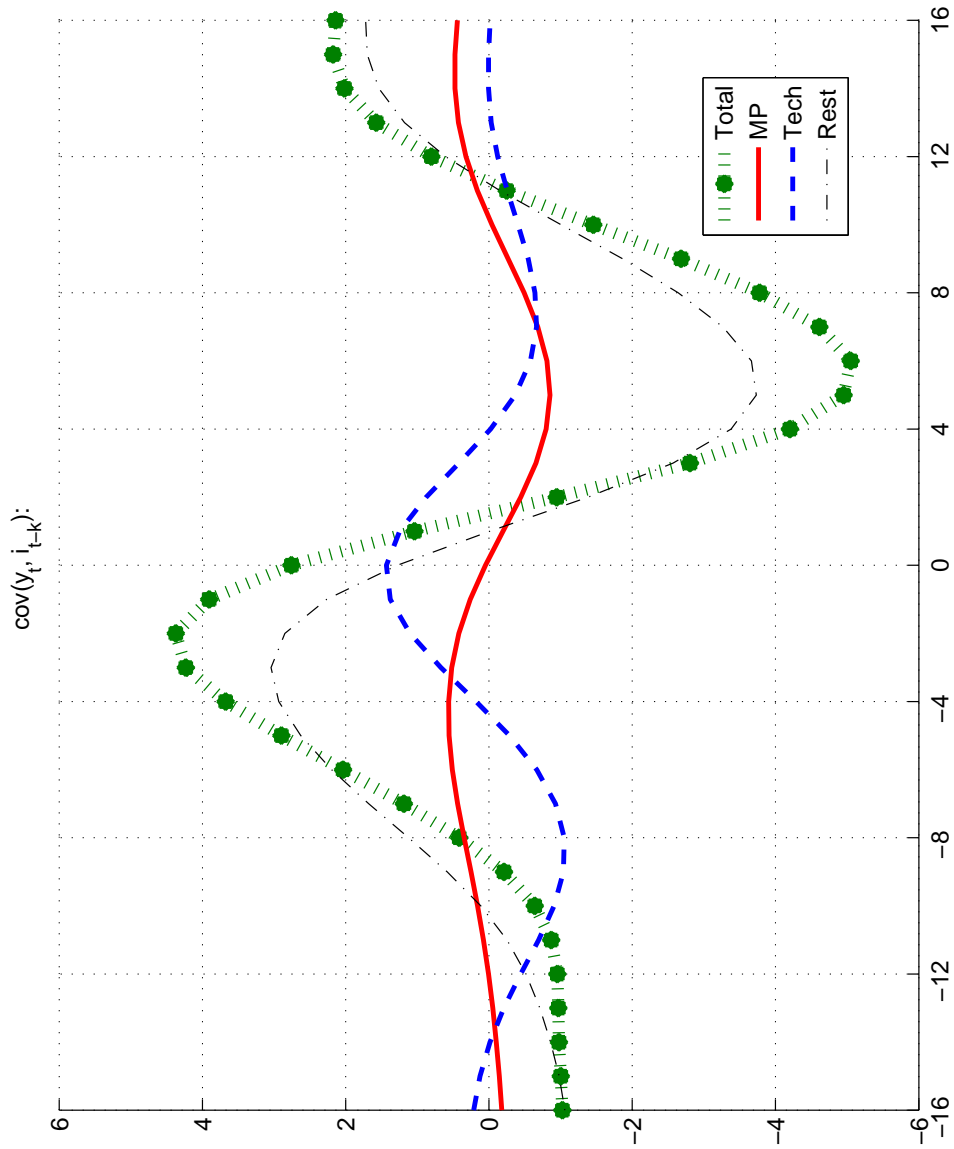
Qualitatively, the sign of the conditional covariances are similar to the ones discussed for the real rate. Conditional on technology shocks, the nominal rate is pro-cyclical and a positive leading indicator for up to 4 quarters. Conditional on monetary shocks, the nominal rate is a-cyclical and a negative leading indicator. Given the small amplitude of these conditional covariances, they are barely significant, but the correlations mostly are, as can be seen from the bootstrapped standard error bands in Figures 3 and 4.

4 Related Literature

To overcome the output-interest rate puzzle, Beaudry and Guay (1996) and Boldrin, Christiano, and Fisher (2001) propose models with habit preferences and frictions to capital accumulation respectively sectoral factor immobility. This matches the real rate evidence by tweaking the transmission mechanism for a single kind of shock, namely technology. But the evidence presented in this study,

²⁹For the pre-Volcker period they find just the opposite and my results would thus be an average over their subsample analysis.

Figure 6: Covariance Decomposition for Nominal Rate and Output



Note: Bandpass filtered moments computed from benchmark VAR described in Section 2. Lags in quarters.

suggests that the standard RBC mechanism for technology works fine.³⁰ It is rather the interaction of several shocks leading to the “puzzling” evidence.

In this spirit, Rotemberg and Woodford (1997) report success with decision lags in a sticky price model³¹. The only structural shock they identify are disturbances to monetary policy. But their solution to the output-interest rate puzzle is based on the interaction with other shocks, which are left unidentified.³² Likewise, Fuhrer and Moore (1995) model the inverted leading indicator property of interest rates with multiple, non-structural shocks and couch their analysis just in terms of unconditional statistics. My paper is an empirical attempt to disentangle the interaction of two candidate shocks in the post-war economy of the U.S.

5 Conclusion

An economic model specifies restrictions on how the economy responds to exogenous forces. Data need not conform to these predictions, either because the specified responses are wrong, or because the set of forces considered in the model

³⁰Beaudry and Guay (1996) recognize the importance of conditioning on technology, too. They use cointegrating properties between output, consumption and investment derived by King et al. (1991), which are similar in spirit to my benchmark specification described in Section 2.1. When conditioning on these permanent shocks, they report negative correlations between output *growth* and the *unfiltered* real rate. Since growth rates amplify high-frequency fluctuations instead of focusing on business cycle characteristics, these results are hardly comparable to my approach and the puzzle framed by King and Watson (1996).

³¹Rotemberg and Woodford (1997) look only at output and the nominal rate. They use linear detrending instead of the stochastic procedures considered here. Still they find similar patterns of covariation and juxtapose their results to the puzzle posed by King and Watson (1996).

³²This is revealed by their impulse response functions (Rotemberg and Woodford 1997, Figure 1). Following a monetary shock, the model’s output responses are negative (respectively zero) at all lags whilst they are positive for the nominal rate. Since conditional lead-lag covariances are just convoluted impulse responses, they are negative (respectively zero) at all leads and lags. This contrasts with the changing signs in the unconditional covariances depicted in my Figure 1 respectively their Figure 2. Rotemberg and Woodford estimate the unidentified drivers for their model from data in order to match the money IRF as good as possible. See also my footnote 31 on their choice of detrending.

does not sufficiently capture those impinging on the real world (or both). Confronted with “puzzling” data, one remedy could be to twist the theory of the economic structure, for instance by using different preferences, frictions etc. But if the standard theory allows for different types of shocks to lead to different responses, it is also natural to investigate whether the “puzzling” patterns in the data arise from a combination of the types of shocks.³³

King and Watson (1996) report an output-interest rate puzzle, because of discrepancies in the *unconditional* correlations of output and interest rates in U.S. data and a variety of calibrated models. But in this context, standard theories already predict different *conditional* correlations for different shocks. My paper attempts to measure what the empirical evidence has to say on these conditional correlations. And indeed, the output-interest rate puzzle appears in a different light, once the bc-statistics are conditioned on structural shocks. At the root of the “puzzle” are not so much the transmission mechanisms of their models, but rather the interaction of several shocks³⁴. Three points stand out:

Conditional on technology shocks, the comovements between output and real rate lines up fairly well with standard models, be it the standard RBC model or the technology channel of textbook New-Keynesian models as studied by Gali (2003), Walsh (2003) or Woodford (2003). For all specifications considered, the

³³A case in point is how Christiano and Eichenbaum (1992) add government spending shocks to RBC theory to resolve the Dunlop/Tarshis/Keynes debate on the overall cyclicity of real wages. See between Dunlop (1938), Tarshis (1939) and Keynes (1939), the issue is also summarized by Sargent (1987, p. 487). Another example for solving the same puzzle with multiple shocks is how Baxter and King (1991) enrich the RBC model with demand shocks.

³⁴Another line of attack in this area has been opened by Dotsey, Lantz, and Scholl (2003) by pointing out that the real rate evidence is sensitive to the choice of price deflator used for constructing the real rate. The widely reported anti-cyclicity of the real rate is particularly strong when deflating with the CPI. In the main text of this paper, the deflator of the personal consumption expenditure index (PCE) is used, which is more consistently measured than the CPI. I can replicate their unconditional findings for other inflation measures with my VAR approach. And the basic results for *conditional* comovements between output and real rate remain valid. A discussion of these results can be found in the web-appendix.

contemporaneous correlation between (bc-filtered) real rate and output is positive. Likewise, the real rate is a positive leading indicator of output for at least a few quarterly lags. When technology shocks are identified using stationary hours, these correlations are also highly significant. Unconditionally, the real rate is widely reported to be just the opposite – namely counter- or a-cyclical and a negative leading indicator. Attempts to match this only with technology shocks appear to be going in the wrong direction.³⁵ The overall behavior must be the outcome of an interaction of several shocks. Indeed:

When conditioning on monetary shocks, the real rate is counter-cyclical and a negative leading indicator as predicted by the simple New-Keynesian models. Such opposing responses to “supply” and “demand” shocks are a general theme in Keynesian models (Bénassy 1995).

Even together, technology and monetary shocks cannot account for the unconditional relationship between output and interest rates. In particular, they cannot explain much for the nominal rate – the big unexplained component coming from inflation. Models need to include other shocks than technology and monetary policy, which are both ignored by my study and the models mentioned above.

³⁵See for instance the RBC modifications of Beaudry and Guay (1996) and Boldrin, Christiano, and Fisher (2001) with habit preferences and frictions like capital accumulation and sectoral factor immobility. Beaudry and Guay (1996) recognize the importance of conditioning on technology. But since they use a quite different detrending method their results of a counter-cyclical real rate even after conditioning on technology are hard to compare with the results in this study. See also footnote 30.

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