Bank Capital and non-verifiable Lending Risk

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Abstract

This paper analyzes the consequences of a flexible demand deposit contract for the need to partly finance a bank with capital when one wants to solve the hold-up problem between the bank and its financiers optimally. It shows that there is no need for bank capital when lending is subject to general, non-verifiable risk. Capital can be helpful, however, when lending risk is intermediary specific.

1 Introduction

Incompleteness of contracts constitutes a major obstacle to project financing. An entrepreneur who cannot commit to the contribution of her project-specific human capital has a strong incentive to hold-up her financiers once the investment is sunk. This incentive stems from the weak bargaining position that financiers have vis-a-vis the entrepreneur. Anticipating the hold-up, financiers may even refuse to finance some otherwise valuable projects. Consequently, there is a tendency to underinvestment (Hart and Moore 1994, Hart 1995).

Intermediated lending through a relationship lender, i. e. a bank, reduces underinvestment. By acquiring project-specific knowledge, the bank gains a better bargaining position vis-a-vis the entrepreneur. It can therefore extract a higher amount from the entrepreneur. Intermediated lending, however, introduces a second hold-up problem. The bank can hold-up its unskilled financiers by threatening to withdraw its project-specific knowledge. The extent to which underinvestment is reduced thus hinges on the extent to which this second problem is solved.

Diamond and Rajan (2000, 2001a, 2001b) argue that demand deposits issued by the bank are an important measure to reduce underinvestment. Deposits deter the bank from holding up financiers. Their disadvantage is, however, that a high level of deposits provokes a bank run when lending is subject to non-verifiable and thus non-contractible risk and the value of the bank‘s assets is relatively low. The resulting loss of relationship lending skills lowers the payment that the financiers get below what the bank could extract from the entrepreneur. It is claimed that the possibility of a bank run potentially gives rise to capital being part of an optimal solution to the underinvestment reduction problem. When the bank is partly financed with capital, bank runs can be prevented. Capital, however, allows the bank to extract rents from capital holders again lowering payments to financiers.

Optimal underinvestment reduction thus trades off the losses due to bank runs and the losses due to rents. The higher the lending risk, the more useful capital is to facilitate lending (Diamond and Rajan 2000).

This paper aims to extend the existing literature on the optimal solution to the hold-up problem between the bank and its financiers. A deposit contract will be introduced which allows some flexibility of payments to depositors and therefore prevents some otherwise inevitable inefficient bank runs when there is non-verifiable lending risk. It will be shown that, as a consequence, capital is a useful measure to reduce underinvestment only when there is a particular type of lending risk.

In the traditional literature, a deposit contract is typically characterized by the following properties: A depositor has the right to request repayment of the deposit‘s face value at any time according to a first come, first served principle. Whenever the bank is reluctant to repay, depositors can force the bank to sell assets with market value equal to the deposit‘s face value and grab the received amount. The value of this option does not depend on the state of the world. Therefore, the bank cannot renegotiate payments to depositors in bad times so that a bank run occurs.

Our deposit contract alters the depositors‘ rights when the bank does not want to or cannot repay. Then, they can seize a prespecified share of the bank‘s assets. The value of this option does depend on the state of the world giving discretion to the bank to negotiate down depositors‘ claims in bad times. In doing so, some inefficient bank runs are prevented. However, discretion is not unlimited so that excessive rent extraction can be prevented, as well.

Subsequently, we analyze two scenarios. In the first, lending risk is general in the sense that the ratio of the ex-ante uncertain maximum amount that the bank can extract to what unskilled financiers can extract from the entrepreneur is constant across states. We will show that in this scenario, deposits can fully solve the hold-up problem between the bank and its financiers. Deposits can ensure that the bank always forwards all it can extract from the entrepreneur to its financiers. That is, deposits can be designed such that they are neither prone to runs nor to rent extraction through the bank. Consequently, financing the bank partly with capital is unnecessary if one wishes to optimally reduce underinvestment. In the second scenario, lending risk is intermediary specific in the sense that the above mentioned ratio of the uncertain maximum amounts is not constant across states. In this setting, demand deposits can no longer fully solve the hold-up problem. This reintroduces capital as a potentially useful instrument to reduce underinvestment when the losses due to runs exceed the losses due to rent extraction.

Our paper thus further narrows the conditions under which capital is part of the optimal solution to the hold-up problem between the bank and its financiers. Provided that deposits are sufficiently flexible, capital is helpful when lending risk is specific to the bank. The roadmap of the rest of this paper is as follows: Section 2 outlines the framework. Section 3 examines the general lending risk case. Section 4 analyzes the intermediary specific lending risk case. Section 5 discusses the results and concludes.

2 The Framework

The following is a generalized version of the one-period framework of Diamond and Rajan (2000). Consider an economy consisting of entrepreneurs and financiers, all being risk neutral. Each entrepreneur is endowed with a production technology. It requires an initial investment of 1 at the beginning of the period. If the entrepreneur contributes her project-specific human capital, the production technology yields a stochastic non-negative cash flow c at the end of the period which depends on the state of the world. States are observable but non-verifiable so that contracts cannot be conditioned on states explicitly. The entrepreneur has no own funds. Therefore, the investment can only be realized when she borrows from financiers. Borrowing is facilitated by a contract that specifies a payment P that the entrepreneur promises to make to financiers.

Contracting is hampered by two factors: First, the entrepreneur cannot commit to the contribution of her project-specific human capital at the beginning of the period. Second, if the entrepreneur withdraws her human capital before the cash flow is realized, the project can be liquidated at best. This yields a stochastic non-negative return ax where $x \in [x_{\min}, x_{\max}]$ and $x \leq c$ holds in every state. The variable a follows a two-point distribution. We assume that $a = 1$ with probability q and $a = \alpha < 1$ with probability $1 - q$. However, ax is available only to a financier who has had close contact to the project from its initiation. As such close contact is time consuming and costly for both, the entrepreneur and the financier, we assume that only one financier, the relationship lender or bank, has access to this liquidation technology. Costs of acquiring relationship lending skills are normalized to zero. Any other financier who liquidates gets only βx with $\beta < \alpha$ and β being constant across states.

In this setting, one can interpret a as a performance measure of the bank which indicates to what extent it succeeds in acquiring project-specific

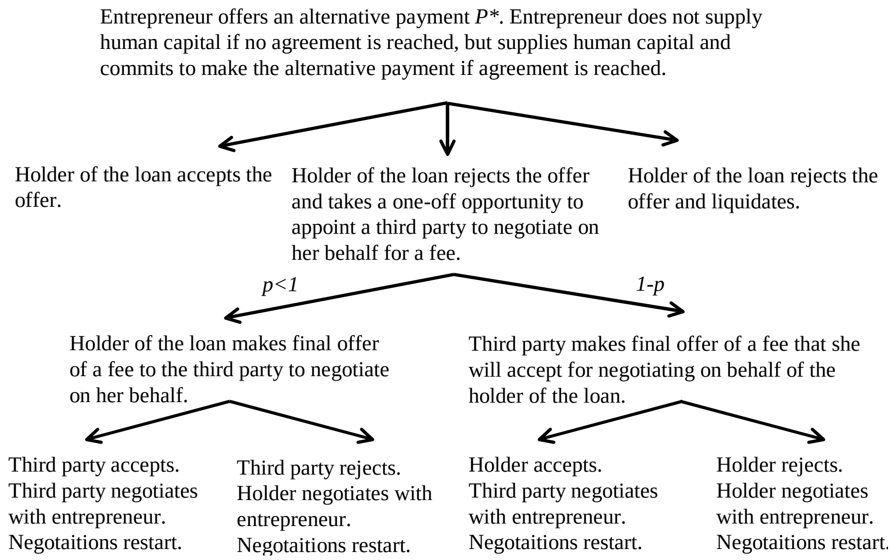

Figure 1: Bargaining between entrepreneur and holder of the loan

knowledge. That is, $a = 1$ stands for a well performing bank while $a = \alpha$ indicates bad performance. Then, x can be understood as some sort of general liquidation value of the project which is available to the bank when it performs good. To keep things simple, we assume that the bank cannot influence q and that x and a are independently distributed. We thus abstract from a bank‘s effort choice and from a potential influence of the general liquidation value on the bank‘s performance.

The combination of the two factors mentioned above potentially gives rise to negotiations after the contract is concluded and the investment is sunk. The entrepreneur can hold-up the party holding the loan by threatening to withdraw her human capital in order to negotiate down a beforehand promised payment. The Negotiations have the following structure: First, the entrepreneur offers an alternative payment P^*. Then, the holder of the loan has three options: (1) She can accept the offer. Then, the entrepreneur contributes her human capital and the holder gets P^*. (2) She can reject the offer and take a one-off opportunity to appoint a third party to negotiate on her behalf for a fee that is fixed as follows. With probability $p < 1$, the appointer makes a take-it-or-leave-it offer to the appointee. With probability $1 - p$, the procedure is vice versa. Then, negotiations restart. (3) She can reject the offer and liquidate. Figure 1 depicts the extensive form of negotiations.

As the entrepreneur can negotiate, there is a maximum payment that a holder of the loan can extract from the entrepreneur. It is equal to the highest pay-off, the holder can get when rejecting the entrepreneur‘s offer within negotiations. Any higher promise of the entrepreneur would result in a threat to withdraw human capital and in negotiations as just described.

The maximum amount that a holder of the loan can extract, taking the realization of x and a as given, can thus be found by checking the negotiation procedure. When the bank holds the loan, it can extract at most ax. When an unskilled financier holds the loan, the respective amount is βx when no bank exists. If, however, a bank exists, the maximum amount that an unskilled financier can extract is equal to the value of appointing the bank to negotiate on her behalf. Similar arguments as in Diamond and Rajan (2000) reveal that this amount is equal to $[pa + (1-p)\beta]\, x$.

In our framework, the probability p measures the bargaining power of an unskilled financier vis-a-vis the bank when she appoints the bank to negotiate on her behalf. Therefore, p indicates to what extent the hold-up problem between the bank and the unskilled financiers is exogenously solved. A higher p implies that the bank extracts more from the entrepreneur on behalf of the unskilled financiers even without a particular contractual arrangement.

The probability q indicates what kind of risk is prevalent. When $q = 1$ (or $q = 0$), there is general lending risk. Then, the ratio of the ex-ante uncertain maximum amount that the bank can extract to what unskilled financiers can extract from the entrepreneur is constant across states and equal to $[p + (1-p)\beta]^{-1}$ (or $\alpha\,[p\alpha + (1-p)\beta]^{-1}$). That is, risk is general when the bank‘s performance is constantly good (or bad). On the contrary, risk is intermediary specific when $q \in (0, 1)$. Then, the ratio is not constant across states and depends on the performance of the bank. When it performs good, $a = 1$, the bank can extract a relatively higher amount from the entrepreneur compared to unskilled financiers than when the performance of the bank is bad, $a = \alpha$. This is due to our assumption $p < 1$. It implies that the amount that unskilled financiers can extract only partially benefits from a good bank performance.

To keep things simple, the bank is assumed to have no own funds. There is no discounting in the economy. Total endowment of financiers exceeds the number of projects available. The best use of endowment besides lending is a storing technology which yields 1 per unit stored. Therefore, financiers agree on lending when they expect to get back at least 1 per unit invested.

The timing of events is as follows: At the beginning of the period the bank concludes a contract with its financiers and the entrepreneur. Then, uncertainty resolves, i. e. the realization of x and a is revealed. Both, x and a are observable but non-verifiable. After that, the bank decides whether to contribute its relationship lending skills or to negotiate with its financiers. Then, the entrepreneur decides whether to contribute her project-specific human capital or to enter into negotiations with whoever holds the loan. At the end of the period cash flows are realized. Then, payments are made.

3 General lending risk

This section concentrates on general lending risk. Therefore, we fix $q = 1$. This implies that $a = 1$ is true with certainty. The bank always performs well in acquiring relationship lending skills. Consequently, the ratio of the maximum amount, that the bank and the unskilled financiers can extract from the entrepreneur, is constant across states. We now show that a deposit contract can fully solve the hold-up problem between the bank and its financiers. This contract ensures that the bank always forwards all it can extract from the entrepreneur to its financiers so that there is no need for bank capital.

The bank concludes a lending contract with the entrepreneur. The latter promises to pay P at the end of the period. As risk is non-verifiable, P cannot be conditioned on the state of the world. If the entrepreneur threats to withdraw her human capital, the bank may liquidate the project. According to the preceding section, this contract implies that given the liquidation value x the bank can extract $\min\{P, x\}$ from the entrepreneur.

With its financiers, the bank concludes a deposit contract which should always prevent bank runs and rent extraction through the bank. Both requirements imply that the amount that the financiers ultimately get should be equal to $\min\{P, x\}$ which is the payment, the bank can extract from the entrepreneur.

The deposit contract that meets both requirements has a total face value D. Each depositor holding a share λ of D may request repayment at any time according to a first come, first served principle. When the bank is reluctant to meet its repayment obligation and tries to negotiate down claims of depositors, a repayment requesting depositor with claim λD may seize a prespecified share $\lambda\gamma$ of the bank's assets, i. e. the bank's loan. In what follows, this contract is represented by the tuple $\langle D, \gamma\rangle$.[1]

[1] The term demand deposit for our contract is appropriate as its main features, with-

The contract that prevents both, bank runs and rent extraction will now be derived in two steps. First, we identify the conditions for prevention of either bank runs or rent extraction. Then, we merge the results and present the optimal contract that meets both requirements.

3.1 Preventing Bank Runs and Rent Extraction

In order to derive the conditions under which a deposit contract $\langle D, \gamma \rangle$ prevents bank runs or rent extraction, one first must determine the payment the bank makes to its depositors given a particular contract $\langle D, \gamma \rangle$ and given that it is not run before the end of the period. This payment can be determined by considering negotiations between the bank and the depositors which are again assumed to follow Diamond and Rajan (2000): First, the bank offers an alternative total face value of deposits D^*. Then, each depositor who holds a share λ of deposits has three options. (1) She can choose not to run the bank and accept the offer. Then, she receives her payment from loans that remain in the bank after completion of the run by other depositors. (2) She can run the bank and join a line with positions allocated by a fair lottery. If there are assets left when she reaches the front of the line, she seizes a share $\lambda\gamma$ of the bank‘s assets, otherwise she gets nothing. (3) She can refuse the offer and negotiate with the bank about who negotiates loans that remain in the bank after the completion of the run by other depositors. The extensive form of negotiations is depicted in figure 2.

For a given liquidation value x, appendix A reveals that in case of the bank negotiating with depositors, the minimum offer D^* that depositors accept without running is $\gamma \min \{P, [p + (1-p)\beta]\, x\}$. When this minimum offer is not lower than D, the bank has no incentive to negotiate down depositors. The bank then contributes its project-specific liquidation skills. Since if it negotiated and offered a total payment $D^* < D$, depositors would run the bank. When, however, this lowest offer falls short of D, the bank enters negotiations with depositors by threatening to withdraw its project-specific liquidation skills. The bank thus pays $\min \{D, \gamma \min \{P, [p + (1-p)\beta]\, x\}\}$ to depositors at the end of the period.[2]

drawal upon request and the first come, first served constraint are commonly associated with deposit contracts (see e.g. Diamond and Dybvig 1983, Diamond and Rajan 2000). The asset seizure clause, however, is rather atypical for deposit contracts. Here, our contract resembles a convertible security that grants the issuer, i. e. the bank, a put option.

[2] There is a second equilibrium when the bank negotiates with depositors. Irrespective of the bank‘s offer, negotiations fail and a run occurs when depositors expect that other depositors do not accept the bank‘s offer and seize assets. Then, it is optimal for them not to accept and seize assets as well. We discuss this in section 5 in detail.

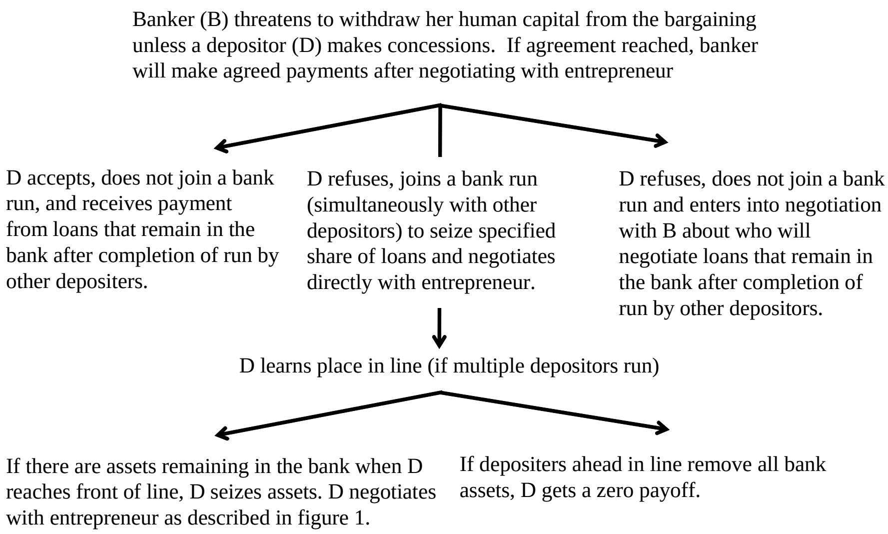

Figure 2: Bargaining between bank and depositors

We have not yet dealt with the bank‘s ability to make payments to depositors. It can do so only if it expects to get from the entrepreneur at least what it plans to pay to depositors. Then, depositors are certain to be paid in full at the end of the period and they have no reason to request repayment before maturity of the project. The opposite is true when the bank expects to get less from the entrepreneur. Then, depositors know that some of them will not get repaid in full in the end if they wait and run the bank. In summary, we can state that bank runs are always prevented if

$$\min\{D, \gamma \min\{P, [p + (1-p)\beta] x\}\} \leq \min\{P, x\} \tag{1}$$

is true for all x. As a consequence, we get:

Lemma 1 *When there is general lending risk, every demand deposit contract $\langle D, \gamma \rangle$ with (a) $D \leq x_{\min}$ or (b) $D \leq P$ and $\gamma \leq [p + (1-p)\beta]^{-1}$ prevents bank runs in any state.*

Thus, there are at least two ways to prevent bank runs. One way is to choose a sufficiently low face value D of deposits so that depositors are always certain to get repaid in full at the end of the period. They then have no incentive to run the bank irrespective of what the value of γ is. That is, a facilitation of negotiations between the bank and the depositors is unnecessary. The other way is to choose a sufficiently low γ, i. e. to let depositors seize a relatively small share of the bank's assets when the bank refuses repayment. Bank runs are then prevented even if the face value D exceeds the amount that the bank can extract from the entrepreneur in some states. This is done by giving the bank enough room to negotiate down depositors' claims when it gets a relatively small amount from the entrepreneur. Note that the two conditions stated in the lemma are sufficient conditions.

With the help of the previous discussion, the conditions for the prevention of rent extraction through the bank can easily be derived. Rent extraction is prevented whenever in every state either the bank is run or the bank pays to depositors exactly what she receives from the entrepreneur. Rent extraction is thus always prevented if

$$\min\{D, \gamma \min\{P, [p + (1-p)\beta] x\}\} \geq \min\{P, x\} \tag{2}$$

is true for all x. This results in:

Lemma 2 *When there is general lending risk, every demand deposit contract* $\langle D, \gamma \rangle$ *with* $D \geq P$ *and* $\gamma \geq [p + (1-p)\beta]^{-1}$ *prevents rent extraction through the bank in any state.*

3.2 Optimal solution to the hold-up problem

Either requirement, prevention of bank runs and prevention of rent extraction, can be met with a suitable deposit contract. Indeed, a brief inspection of the two lemmas reveals that one contract meets both requirements for all states:

Proposition 1 *When there is general lending risk, the demand deposit contract* $\left\langle P, [p + (1-p)\beta]^{-1} \right\rangle$ *prevents bank runs and rent extraction through the bank. It fully solves the hold-up problem between the bank and its financiers.*

The deposit contract specified in proposition 1 facilitates negotiations between the bank and its depositors in bad times only. The bank does not extract rents. Bank runs are fully avoided. The bank always forwards everything, it can extract from the entrepreneur, to its depositors. The contract thus endogenously fully solves the hold-up problem between the bank and its financiers. This is true although risk is non-verifiable in the framework so that state contingent contracting is impossible. The share of assets that a depositor may seize when the bank tries to negotiate is indeed fixed and therefore independent of states. The value that seizing assets has to depositors, however, does depend on the state of the world. Consequently, depositors make no concession vis-a-vis the bank in good times when the bank receives a relatively high amount from the entrepreneur. They make, however, concession in bad states when the bank is negotiated down by the entrepreneur. In this case, negotiations between the bank and its depositors can be facilitated, so that no bank run needs to occur. State contingent payments from the bank to its depositors are thus feasible.

One might expect that when the bank is negotiated down by the entrepreneur, it uses negotiations with depositors to extract rents. This does not happen when there is general lending risk. Then, the ratio of what the bank receives from the entrepreneur to what depositors can extract from the entrepreneur is constant across states. The same is true for the ratio of what depositors can extract to what the bank offers to depositors within negotiations. It is equal to γ^{-1}. Consequently, the ratio of what the bank receives to what the bank offers is constant across states. This ratio depends on γ. Provided that γ is suitably fixed, it is equal to one. This ensures that the bank cannot use negotiations with depositors to extract rents. The hold-up problem between the bank and financiers is thus fully solved.

The implications of this result for the underinvestment reducing effect of the bank are straightforward. Both, bank runs and rent extraction through the bank can be prevented when there is general lending risk. Therefore, underinvestment can maximally be reduced in the sense that any entrepreneur who can promise a sufficient payment to the bank has access to financing. This is due to the fact that it is possible to let the bank forward all it receives from the entrepreneur to the financiers. Therefore, it is possible to convince the financiers to provide the funds for the investment.

4 Intermediary specific lending risk

In this section, we analyze intermediary specific risk. Therefore, we let $q \in (0, 1)$ so that the performance of the bank with respect to acquiring project-specific knowledge is good with probability q and bad with probability $1-q$. It will be shown that in this setting, the hold-up problem between the bank and its financiers cannot be fully solved. One cannot simultaneously prevent bank runs and rent extraction. This introduces bank capital as a potentially useful instrument to finance the bank.

As we are interested in underinvestment reduction, we concentrate on a lending contract that maximizes payments of the entrepreneur, i. e. we set $P = x_{\max}$. This implies that as long as no run takes place the bank always receives the maximum of what it can extract from the entrepreneur. It gets x when performing well and αx when performing badly.

We proceed in the same way as before. First, we derive conditions for bank run and rent extraction prevention. Then, we discuss the optimal solution to the hold-up problem between the bank and financiers.

4.1 Preventing Bank Runs or Rent Extraction

We begin with the conditions ensuring the prevention of bank runs. When the bank‘s performance is good, the earlier obtained results apply. Then, a bank run is prevented as long as for $P = x_{\max}$, equation (1) holds for all liquidation values x. When the bank performs badly, the arguments are rather similar to those that led to equation (1). Given the lending contract with $P = x_{\max}$, the bank receives αx from the entrepreneur provided that it holds the loan at the end of the period. According to appendix A, the bank then pays $\min\{D, \gamma[p\alpha + (1-p)\beta]x\}$ to its depositors. As long as the latter amount never exceeds the former, a bank run is prevented when the bank performs badly. Putting together both cases results in the relevant condition. A bank run is always prevented when

$$\min\{D, \gamma[p\alpha + (1-p)\beta]x\} \leq \alpha x \qquad (3)$$

is true for all x.

Lemma 3 *When lending risk is intermediary specific, every demand deposit contract $\langle D, \gamma \rangle$ with (a) $D \leq \alpha x_{\min}$ or (b) $\gamma \leq \alpha [p\alpha + (1-p)\beta]^{-1}$ prevents bank runs in any state.*

The set of contracts satisfying the conditions stated in the lemma, which are both necessary and sufficient for bank run prevention, is depicted in figure 3. Note that now, the restrictions placed on the contracts are somewhat tighter than for the general lending risk case as presented in lemma 1. This is true for both, γ and the face value D. This is due to the possible occurrence of a bad bank performance.

The conditions for rent extraction prevention must also be derived for both cases, a good and a bad bank performance. When the bank performs well, we can again use the results obtained earlier. The relevant equation is (2) for $P = x_{\max}$. When it performs badly, parallel arguments reveal that the bank pays what it gets or is run if $\min\{D, \gamma[p\alpha + (1-p)\beta]x\}$ is no lower than αx in all states. As this is true as long as (2) is true, we can conclude that rent extraction is always prevented when

$$\min \{D, \gamma [p + (1-p) \beta] x\} \geq x \tag{4}$$

is true for all x. This implies:

Lemma 4 *When lending risk is intermediary specific, every demand deposit contract $\langle D, \gamma \rangle$ with $D \geq x_{\max}$ and $\gamma \geq [p + (1-p) \beta]^{-1}$ prevents rent extraction through the bank in any state.*

The set of contracts satisfying the conditions for rent extraction prevention is also depicted in figure 3. This set is basically identical to the respective set for the general lending risk case (see lemma 2). A check of lemma 3 and 4 reveals that the set of contracts preventing bank runs is disjoint from the set preventing rent extraction. That is, there exists no deposit contract $\langle D, \gamma \rangle$ that simultaneously prevents bank runs and rent extraction when there is intermediary specific risk. It is thus impossible to fully solve the hold-up problem between the bank and its financiers. The rationale behind this result is straightforward. When risk is intermediary specific, the ratio of what the bank can extract from the entrepreneur to what depositors can extract is not constant across states. The ratio then depends on the bank's performance. It is small for $a = \alpha$ when the bank's skills are relatively bad compared to depositors and it is large for $a = 1$ when the bank's skills are relatively good. That is, the amount the bank can extract per unit of what depositors can extract is lower for $a = \alpha$ than for $a = 1$. However, the ratio of what depositors can extract from the entrepreneur to what the bank offers to pay within negotiations with depositors is constant across states. It is equal to γ^{-1}. Consequently, the ratio of what the bank receives to what it offers is not constant across states. It varies with the realization of a. It is lower for $a = \alpha$ than for $a = 1$. This is true irrespective of which γ is chosen. Therefore, it is impossible to design a deposit contract that ensures that the latter ratio is equal to one for all states. This, however, would be necessary in order to prevent both, bank runs and rent extraction.

The problem is thus that across states, payments to depositors can vary only in line with the amount that depositors can extract from the entrepreneur. Whenever this amount does not vary in line with the amount that the bank can extract, the hold-up problem between the bank and its depositors cannot be fully solved by the demand deposit contract.

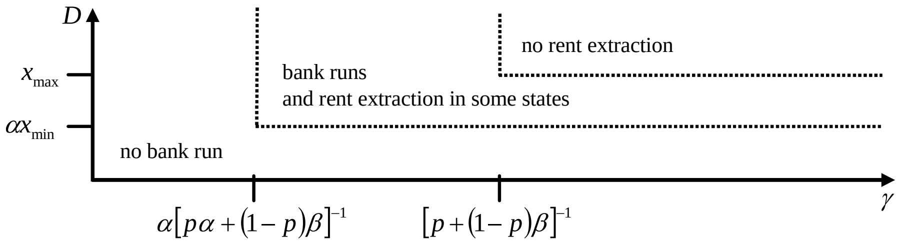

Figure 3: Consequences of different contracts

4.2 Optimal solution to the hold-up problem

The previous section has revealed that when there is intermediary specific risk, one cannot prevent both, bank runs and rent extraction. One must choose between either of the two targets or forgo both. We now ask which financial structure optimally solves the hold-up problem between the bank and financiers by maximizing expected payments to financiers, and therefore maximally reduces underinvestment.

Let us first investigate contracts that prevent bank runs. They are necessarily associated with rent extraction through the bank. There is, however, a way to minimize rents, by choice of an adequate deposit contract plus issuance of a capital. We follow Diamond and Rajan (2000) in understanding the latter as a long term claim without a first come, first served right to cash flows. Capital is a claim on the residual value of the bank after depositors are paid out. If the bank tries to negotiate with capital holders, they have the right to seize assets in full but then they become fully responsible for depositors‘ claims.[3]

What do depositors and capital holders receive given the realization of x and a and a particular financial structure that prevents bank runs? Appendix B shows that the total amount that the bank‘s financiers get is

$$\begin{aligned} &pax + (1-p)\min\{D, \gamma[pa + (1-p)\beta]x\} \\ &\quad + (1-p)\max\{\beta x - \min\{D, \gamma[pa + (1-p)\beta]x\}, 0\}. \end{aligned} \tag{5}$$

As this is weakly increasing in both, D and γ, it follows that:

[3]Capital is thus a claim similar to outside equitity (see e.g. Aghion and Bolton 1992, Hart 1995 and Myers 2000).

Lemma 5 *When lending risk is intermediary specific and one wishes to prevent bank runs in any state, expected payments to financiers are maximized when the bank issues demand deposits* $\left\langle x_{\max}, \alpha \left[p\alpha + (1-p)\beta\right]^{-1}\right\rangle$ *plus capital.*

With a mixed financial structure as described in the lemma, depositors get all that the bank can extract from the entrepreneur when the bank performs badly, $a = \alpha$. The bank then does not extract rents and capital holders receive a zero pay-off. When, however, the bank performs well, $a = 1$, depositors get less than what the bank receives implying a rent for the bank and a positive pay-off for capital holders.

Now, we turn to contracts that prevent rent extraction which are somewhat easier to analyze. As the bank pays to depositors what it gets or is run, there is no use for bank capital. Depositors are thus the only financiers of the bank. Payments to them should be maximized. A check of lemma 4 and equation (3) reveals that bank runs cannot be prevented when the bank performs badly, $a = \alpha$. Then, the bank is run and depositors receive $\left[p\alpha + (1-p)\beta\right] x$ only. Bank runs can thus only be prevented when the bank performs good. Consequently, we get:

Lemma 6 *When lending risk is intermediary specific and one wishes to prevent rent extraction in any state, expected payments to financiers are maximized when the bank issues demand deposits* $\left\langle x_{\max}, \left[p + (1-p)\beta\right]^{-1}\right\rangle$ *only.*

The deposit contract specified in lemma 6 ensures that a well performing bank always forwards all it can extract from the entrepreneur to its depositors. If, however, it performs badly, the bank is run.

Thus far, we have not discussed deposit contracts not included in the sets specified in lemma 3 and 4. They are associated with both, bank runs and rent extraction in some states. Therefore, they can be issued together with bank capital. Appendix C reveals that an in-depth discussion of these contracts is unnecessary as they never maximize payments to financiers.

There are thus two candidate financial structures for optimally solving the hold-up problem. The first candidate consists of demand deposits plus bank capital as described in lemma 5. It prevents bank runs but implies rents for the bank when $a = 1$. The second consists only of deposits (see lemma 6). It prevents rents for the bank but implies runs when $a = \alpha$. A comparison of the respective expected pay-off of financiers results in:

Proposition 2 *When there is intermediary specific lending risk, a mixed capital structure is the best solution to the hold-up problem between the bank and its financiers only if*

$$q \leq \frac{(\alpha - \beta)\,[p\alpha + (1-p)\,\beta]}{(\alpha - \beta)\,[p\alpha + (1-p)\,\beta] + (1-p)\,(1-\alpha)\,\beta} =: q^{crit}.$$

Otherwise, the best solution is achieved by issuance of deposits only.

Intermediary specific risk thus potentially introduces capital as a component of the bank‘s financial structure. Capital helps to solve the hold-up problem when the probability q, that the bank performs good, is sufficiently low so that the event $a = 1$ is sufficiently improbable. The reason is instructive. There are two options available, both having a negative impact on payments to financiers. One can opt for rents when the bank performs good or one can opt for runs when the bank performs badly. When q is below some threshold q^{crit}, it is sufficiently improbable that the bank‘s performance is good. Therefore, potential rent extraction is sufficiently unlikely to occur, its negative impact is minor compared to the impact of bank runs. Consequently, a mixed financial structure provides the best solution to the hold-up problem as this prevents runs when the bank performs badly. When q is large, the opposite is true. Then it pays more to prevent rents. Consequently, a financial structure solely consisting of deposits is most attractive.

Parameter changes have an effect on the critical value of q. An increase in α implies that the ratio of what the bank can extract to what depositors can extract increases for $a = \alpha$. As the ratio is constant for $a = 1$, the two ratios converge when α increases. As a consequence, if the bank extracts rents, it can do so to a decreasing extent. This makes bank run prevention relatively more attractive, i. e. q^{crit} increases. Basically the same argument applies when β decreases or p increases. This leads again to a convergence of the two ratios making rent extraction less painful when it occurs. Note that when α approaches 1 or β approaches zero, the economy converges to the case that we examined in the preceding section. Consequently, the rent extraction problem diminishes so that q^{crit} approaches 1. For α approaching β, the rent extraction problem becomes predominant so that q^{crit} approaches zero.

Bank capital is thus a useful measure to solve the hold-up problem between the bank and financiers when two conditions are met. Lending is subject to observable but non-verifiable intermediary specific risk and the loss due to rent extraction is lower than the loss due to bank runs.

Consequently, the underinvestment reducing effect of the bank is somewhat limited when there is intermediary specific risk. Some entrepreneurs cannot finance their project although they could promise a sufficient payment to the bank as the loss due to runs or rents is too high to let financiers agree on lending.

5 Discussion

This paper has shown that a deposit contract exists which allows for some flexibility of payments to depositors. As a consequence, bank capital potentially helps to reduce underinvestment only if lending risk is intermediary specific, i. e. if payments from the entrepreneur to the bank do not vary sufficiently in line with what unskilled financiers could extract from the entrepreneur. Flexibility of deposits is feasible even if contracts cannot be conditioned on risk explicitly. When the bank tries to renegotiate payments, one can give depositors the right to seize a fixed share of assets. The value of this right depends on the state of the world. It is high in good times and it is low in bad times. Therefore, depositors make concessions vis-a-vis the bank in bad times only. Inefficient bank runs can thus be prevented. This prevention makes our contract dominate the deposit contract traditionally analyzed in the literature, which gives depositors the right to force the bank to sell assets and grab the received amount when the bank tries to renegotiate payments. As the value of this option does not depend on the state of the world, depositors make no concessions in bad times and run the bank.[4]

Throughout the analysis, we assumed that, when the bank negotiates with depositors, it always succeeds in reaching an agreement, provided that this is possible. There is, however, a second equilibrium which we ignored thus far. When the bank negotiates, a depositor does not accept any offer from the bank provided that she expects other depositors not to accept as well. The consequence then is a bank run. The existence of multiple equilibria is not specific to the negotiations game. It is common to all types of deposit contracts so that the question of equilibrium selection arises. In this paper, we have applied the Pareto-criterion to this problem. This can be defended by a self selection argument. Any financier who doubted that

[4]Note, that our description of the traditional deposit contract differs from the description given in Diamond and Rajan (2000). In their view, it is possible that the value of both rights, the right to force the bank to sell assets and the right to seize assets, is equal to the deposit‘s face value. This, however, is impossible for the latter right when risk is non-verifiable since the share of the assets that may be seized by a depositor would then have to vary with the state of the world implying state contingent contracting.

perfect coordination is possible would not agree on the respective financial contract. She would therefore not provide funds to the bank. It should be noted, however, that there are other equilibrium selection mechanisms which would imply at least a positive probability of coordination failure.[5]

Even is, however, one attaches a positive probability to coordination failure in case of negotiations, our main results continue to hold as long as this probability is not too high. Then, it is still true that capital emerges as part of the optimal solution to the hold-up problem between the bank and financiers only when there is intermediary specific lending risk.

Appendix A

This appendix derives the lowest offer that depositors accept within negotiations with the bank. To keep the exposition simple, we assume that the bank issues deposits to m financiers with m being large so that we avoid integer problems and we assume that each financier holds a total claim $\frac{1}{m}D$, i. e. $\lambda = \frac{1}{m}$. Both simplifications are not crucial and could easily be relaxed without altering the results.

As a preliminary step, we determine the value that seizing a share $\frac{\gamma}{m}$ of the bank‘s assets has to a single depositor who holds a share $\frac{1}{m}$ of deposits. We know from section 2 that depositors as unskilled lenders can extract at most $[pa + (1-p)\beta]x$ from the entrepreneur. Therefore, given the lending contract, they would receive $\min\{P, [pa + (1-p)\beta]x\}$ from the entrepreneur. Consequently, the value of seizing a share $\frac{\gamma}{m}$ of the loan is $\frac{\gamma}{m}\min\{P, [pa + (1-p)\beta]x\}$ for a depositor who holds a share $\frac{1}{m}$ of deposits.

Now, consider negotiations between the bank and depositors as depicted in figure 2. Suppose the bank makes an offer $D^* \leq \min\{P, ax\}$. Besides accepting, the best action for a depositor is to join a bank run. She then expects get a share $\frac{\gamma}{m}$ of the bank‘s assets with probability

$$\Psi = \begin{cases} 1 & \text{if } \frac{m}{\gamma} \geq E(\tau) \\ \frac{\frac{m}{\gamma}}{E(\tau)} & \text{if } \frac{m}{\gamma} < E(\tau) \end{cases},$$

where $\frac{m}{\gamma}$ is the number of running depositors necessary to fully disintermediate the bank and $E(\tau)$ is the expected number of running depositors. Ψ is thus the probability that the bank is not fully disintermediated before the

[5] An overview over some of the criteria commonly used in economics can be found in Cooper (1999). There is a vast literature on equilibrium selection based on Diamond and Dybvig‘s (1983) model, see e. g. Ennis (2003), Peck and Shell (2003) or Temzelides (1997).

depositor reaches the front of the line. With probability $1-\Psi$, she expects to get nothing. Consequently, the expected value of joining a bank run, given a, x and $E(\tau)$, is

$$\frac{\gamma}{m}\Psi \min\{P, [pa + (1-p)\beta] x\} \tag{6}$$

for the depositor. The expected value of accepting the offer is

$$\frac{1}{m}\min\left\{\max\left\{0, \left(1-\frac{\gamma}{m}E(\tau)\right)\min\{P, ax\}\right\}, D^*\right\}. \tag{7}$$

There is an equilibrium in the negotiations game where all depositors accept the bank‘s offer when (7) is not lower than (6) given that expectations of each depositor are consistent with the equilibrium actions, i. e. $E(\tau) = 0$. Thus, as long as the bank offers at least $D^* = \gamma \min\{P, [pa + (1-p)\beta] x\}$, there is an equilibrium in which depositors accept the offer and decide not to run the bank.

Appendix B

This appendix derives the total amount that the bank‘s financiers get when it opts for a mixed financial structure. We begin with payments from the bank to capital holders which depend on the value seizing assets has to them.

Suppose that capital holders have seized the loan. Consider negotiations between the entrepreneur and capital holders as given in figure 1. Suppose the entrepreneur has made an offer. The capital holders can accept the offer, liquidate, or they can appoint the bank to negotiate on their behalf for a fee. Capital holders may offer the fee with probability p. They offer the smallest fee that the bank accepts. This fee is equal to zero so that the bank extracts ax on behalf of the capital holders. Their surplus is thus $ax - \min\{D, \gamma [pa + (1-p)\beta] x\}$ as they are responsible for paying out depositors. With probability $1-p$, the bank may offer the fee. The bank chooses the fee such that capital holders‘ surplus is equal to $\max\{\beta x - \min\{D, \gamma [pa + (1-p)\beta] x\}, 0\}$, which is their surplus if no agreement is reached between capital holders and the bank. The expected surplus of appointing the bank to negotiate on behalf of capital holders is equal to what the entrepreneur offers when negotiating with capital holders. And this in turn is equal to what the bank offers when negotiating with capital holders.

As bank runs are prevented, depositors get $\min\{D, \gamma [pa + (1-p)\beta] x\}$. Adding what capital holders get yields after some rearrangements (5).

Appendix C

We prove the suboptimality of contracts not included in Lemma 3 and 4 is two steps.

First, we show that the bank does not maximize payments to financiers when it issues bank capital plus demand deposits $\langle D, \gamma \rangle$ with either

$$D \geq x_{\max} \text{ and } \gamma \in \left(\alpha \left[p\alpha + (1-p)\,\beta \right]^{-1}, \left[p + (1-p)\,\beta \right]^{-1} \right) \tag{8}$$

or

$$D \in (\alpha x_{\min}, \alpha x_{\max}) \text{ and } \gamma > \alpha \left[p\alpha + (1-p)\,\beta \right]^{-1}. \tag{9}$$

When the bank issues capital plus deposits as specified in either (8) or (9) the expected total payment to the bank‘s financiers is obviously strictly smaller than $[q + (1-q)\,\alpha - (1-q)\,(1-p)\,(\alpha - \beta)]\, E(x)$, which is the expected payment that the financiers receive when the bank uses the contracts specified in lemma 6.

Second, we show that the bank does not maximize payments to financiers when it uses capital plus demand deposits $\langle D, \gamma \rangle$ with

$$D \in (\alpha x_{\min}, \alpha x_{\max}] \text{ and } \gamma > \alpha \left[p\alpha + (1-p)\,\beta \right]^{-1}. \tag{10}$$

Note that within this set of contracts, the bank maximizes payments to its financiers when it fixes $\gamma = [p + (1-p)\,\beta]^{-1}$. Therefore, we restrict attention to deposit contracts satisfying this property. Suppose the bank issues capital plus demand deposits $\left\langle \alpha\overline{x}, [p + (1-p)\,\beta]^{-1} \right\rangle$ with $\overline{x} \in (x_{\min}, x_{\max}]$. These contracts imply higher payments to financiers than the contracts consistent with lemma 5, if

$$q \geq \frac{(\alpha - \beta)\, E(x) - \int_{\overline{x}}^{x_{\max}} (\alpha\overline{x} - \beta x)\, f(x)\, dx}{\Omega - \beta E(x)} =: q_1^{crit}.$$

This condition is necessary but not sufficient. These contracts imply higher payments to financiers than the contracts consistent with lemma 6, if

$$q < \frac{\int_{\overline{x}}^{x_{\max}} (\alpha\overline{x} - \beta x)\, f(x)\, dx}{E(x) - \Omega} =: q_2^{crit}$$

with

$$\Omega = \int_{x_{\min}}^{\max\{x_{\min}, \alpha\overline{x}\}} x f(x)\, dx + \int_{\max\{x_{\min}, \alpha\overline{x}\}}^{\overline{x}} \alpha\overline{x} f(x)\, dx + \int_{\overline{x}}^{x_{\max}} \beta x f(x)\, dx.$$

This condition is both, necessary and sufficient. By rearranging terms, it can be shown that $q_1^{crit} > q_2^{crit}$. Thus, there exists no q where capital plus demand deposits consistent with (10) maximizes payments to financiers.

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