Why is international risk-sharing so low despite remarkable development of international capital markets?

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Abstract

Low and mostly negative correlations of real exchange rates and aggregate consumption ratios are considered as evidence for low international risk-sharing (Backus and Smith (1993)), which however is surprising given the consistent development of international asset markets that has taken place in the last decades. This paper shows that the observed degree of international risk-pooling could be explained by the presence of distribution services combined with country's openness in an incomplete asset market framework. After a shock occurs, exchange rate-based readjustments of real exchange rates and consumption ratios take place which run counter the adjustments that are consistent with optimal risk-sharing patterns. The interaction of these effects generates comovement that can be either negative or positive conditional on the shock. Due to this mechanism the model predicts low (unconditional) correlations consistent with the evidence.

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1 Introduction

Modeling domestic and international asset markets by means of Arrow-Debreu complete asset frameworks has become a standard practice in international Macroeconomics. The question is how well capital markets can be approximated by this framework. The strategy that has been adopted is to test whether implications from complete asset markets are fulfilled in the data.

The most obvious implication of complete asset markets is that country-specific shocks should be significantly pooled. In their famous paper Backus and Smith (1993) show that optimal international risk-sharing in a world that is characterized by large deviations from purchasing power parity, would imply that domestic households should consume more when their consumption basket is relatively cheap. In other words, optimal risk-sharing would imply that real exchange rates and consumption ratios are positively correlated. This prediction plays however havoc with empirical evidence: for OECD countries this correlation is low and in most of the cases even negative. See in addition to Backus and Smith (1993), also Kollman (1995), Ravn (2001) and Corsetti, Pesenti, and Leduc (2004).

According to Obstfel and Rogoff (2001) *"incompleteness of asset markets is the major reason"*[1] why changes in the real exchange rate and in the consumption ratio are not positively correlated in practice. However, even though asset markets with state-contingent Arrow-Debreu securities might be a (too) ideal representation, international asset markets have as a matter of fact experienced a large development in the last decades. Thus, the question is why despite this development movements of real exchange rates and consumption ratios do not seem to be consistent with risk-sharing patterns.

Given that implications from complete markets are clearly rejected by the empirical evidence, one would expect that relaxing the assumption of complete markets would automatically allow a model to generate patterns that are closer to the evidence. However, this is not necessarily the the case (see Obstfel and Rogoff (2001) for a discussion). In fact, Chari, Kehoe, and McGrattan (2002) develop a model which neither under complete markets nor

[1] Obstfel and Rogoff (2001).

under incomplete markets can match the negative correlation between real exchange rates and relative consumptions.

Recently, Corsetti, Pesenti, and Leduc (2004) have made substantial progress in resolving the puzzle. They propose a model driven by productivity shocks that is calibrated to the US economy and an aggregate of OECD countries and show that it can generate a real exchange rate-consumption ratio correlation (hereinafter Backus-Smith correlation) that is close to the one found in the data, when price elasticity of imports is low enough due to the presence of distribution services.

This paper also features distribution services, but adopt a different approach to investigate the reasons for the large departures from optimal risk-sharing. The starting point is that empirical correlations are the result of movements which might differ in a significant way depending on what source they have been generated[2]. For this reason, this paper models an economy in which there are multiple sources (productivity and monetary) of fluctuations, and investigates comovement of real exchange rates and consumption ratio conditional on each source. The approach consists thus in analyzing how the various conditional comovements interact in generating unconditional comovement pattern. This analysis is performed both under complete and incomplete asset markets. Furthermore, to highlight peculiarities of Backus-Smith comovement, the decomposition analysis is used to investigate also two other kind of comovements. Empirical evidence also provide examples of correlations that, unlike the one of real exchange rate and relative consumption, are very high. Two well-known examples are the correlation of real and nominal exchange rates and the correlation of term of trade and nominal exchange rate (see e.g. Chari, Kehoe, and McGrattan (2002), Obstfel and Rogoff (2000)).

The paper builds on general equilibrium two-country model in which the only nominal rigidity is that of wages as in Obstfel and Rogoff (2000) and firms engage in international price discrimination *à* la Corsetti and Dedola (2005). The model allows for both monetary and technology shocks driving forces. However, differently from the usual Balassa-Samuelson-

[2]For instance Gali (1999) and Christiano and Eichebaum (1992) show that comovement of hours and labor productivity is either positive or negative depending on what shock it has been generated.

based approach that only distinguishes between productivity shocks in the tradable and nontradable sectors, the model also consider productivity shocks in the distribution sector. Recent evidence emphasizes in fact that distribution sector has been characterized by a substantial and rapid productivity (see e.g. Pilat (1997) and Bradford (2005)), which contrasts with the low productivity improvement of all other the nontradable sectors[3]. Finally, the model allows for the use of distribution services at different stages of the production-consumption chain.

The paper is organized as follows. Section 2 discusses the mechanism that leads to perfect risk-sharing and thus perfect positive comovement of real exchange rate and relative consumption in the complete market case. Section 3 discusses how the mechanism changes under incomplete asset markets. Section 4 models the production side of the economy and the various technology shocks. Section 5 proposes a framework to analyze comovement of the variables conditional on a given shock. Section 6 discusses how conditional and unconditional comovements differ under complete and incomplete asset markets. Section 7 investigates the role played in this context by distribution services and country's openness. Section 8 summarizes and concludes.

2 Lessons from the complete asset market case

Let's consider two countries, a first country denominated as the *home* country and a second denominated as the *foreign* country. In their seminal paper Backus and Smith (1993) emphasize that consumption risk is optimally hedged between these countries when the following allocation is achieved:

$$qU_C = U^*_{C^*} \tag{1}$$

where $q = \varepsilon P^*/P$ is the real exchange rate, ε is the nominal exchange rate, P and P^* are the domestic and foreign consumer price indices, and $U^*_{C^*}$ and U_C are the marginal utilities

[3]This has been the motivation for the investigation of distribution sector productivity undertaken in some recent papers (see for instance MacDonald and Ricci (2003) and Devereux (1999)).

of consumption for the domestic and foreign households, respectively. Backus and Smith point out that when the functional form for the utility function is separable in its arguments then (1) implies a positive correlation between changes in the real exchange rate q and in the consumption ratio $\frac{C}{C^*}$.

Chari, Kehoe, and McGrattan (2002) show that when asset markets are complete, in the sense that there is a set of state-contingent securities available to consumers, a decentralized equilibrium delivers this internationally optimal allocation. To see that, assume that in each period of time t, an economy experiences one of finitely many states, s_t. Let $s^t = (s_0, s_1, ..., s_t)$ denote the history of events up through period t. The probability of any particular history s^t is denoted $\pi(s^t)$. $S(s^t, s_{t+1})$ denotes the holdings of the state-contingent security which is purchased in t given s^t and which delivers 1 unity in currency of country h if state s_{t+1} occurs.

The representative consumer in the home country faces the following sequence of budget constraints:

$$\begin{aligned} & M(s^t) + \textstyle\sum_{s^{t+1}} Q(s^{t+1}|s^t) S(s^{t+1}) \\ & \leqslant M(s^{t-1}) + S(s^t) + w(s^t)L(s^t) + \Pi(s^t) - P(s^t)C(s^t) - T(s^t) \end{aligned} \tag{2}$$

where $M(s^t)$ and $L(s^t)$ denote nominal money balances and labor, respectively. $C(s^t)$ is a consumption aggregate that includes all domestic and (imported) foreign goods consumed by the domestic consumer. $P(s^t)$, $w(s^t)$ and $\Pi(s^t)$ denote consumption aggregate price, real wage rate and the profit, respectively. $Q(s^{t+1}|s^t)$ is the price of the state-contingent securities in units of home currency t given s^t.

Domestic consumers choose consumption, money balances, labor and securities holdings to maximize their lifetime utility:

$$\mathcal{U} = \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t) \beta^t U \left(C(s^t), \frac{M(s^t)}{P(s^t)}, L(s^t) \right) \tag{3}$$

Seignorage revenue is reimbursed to households in a lump-sum fashion and hence the government budget constraint is given by:

$$M(s^t) - M(s^{t-1}) = T(s^t) \tag{4}$$

The monetary authority is assumed to credibly commit to monetary rules. The optimization problem of domestic and foreign consumers imply that (see Appendix 1 for details):

$$q(s^t) = \zeta \frac{U^*_{C^*}(s^t)}{U_C(s^t)} \tag{5}$$

Since ζ is a constant that depends of initial conditions and that can be set equal 1 without any loss of generality (see Chari, Kehoe, and McGrattan (2002)), it follows that when asset markets are complete the internationally efficient allocation (1) can be achieved in a decentralized equilibrium. Moreover, by assuming the following standard functional form for the utility function (see e.g. Obstfel and Rogoff (2000) and Corsetti and Pesenti (2005)):

$$U(s^t) = \log C(s^t) + \chi \log \left(\frac{M(s^t)}{P(s^t)} \right) - \kappa L(s^t) \tag{6}$$

it follows that complete asset markets imply a perfect, positive comovement between the real exchange rate and aggregate consumption ratio:

$$q(s^t) = \frac{C(s^t)}{C^*(s^t)} \tag{7}$$

The point now is to clarify the mechanism that generates this result. Let's define $\mu(s^t) \equiv P(s^t)C(s^t)$ and $\mu^*(s^t) \equiv P^*(s^t)C^*(s^t)$. The variables μ and μ^* can be conveniently used to to characterize monetary policy (see Corsetti and Pesenti (2005) and Corsetti and Dedola (2005)). In fact, in a symmetric equilibrium we have that:

$$\mu(s^t) = \frac{M(s^t)}{\chi} \left(\frac{Q(s^t) - Q(s^{t+1})}{Q(s^t)} \right) \tag{8}$$

which implies that given a time path for $\mu(s^t)$, there is a corresponding sequence of Home money stock. Thus, a monetary expansion in period t and history s^t is associated with a higher $\mu(s^t)$. Then (7) can be rewritten as:

$$\varepsilon(s^t) = \frac{\mu(s^t)}{\mu^*(s^t)} \tag{9}$$

Equations (7) and (9) suggest that positive comovement of real exchange rate is associated with a particular process of exchange rate determination, namely one in which the equilibrium exchange rate is exclusively determined by contemporaneous monetary policy of the domestic and foreign country. This characteristic of complete asset markets is quite robust to model specification (see e.g. Devereux and Charles (2001) and Gali and Monacelli (2005)). In particular, this finding can be generalized with respect to two viewpoints. First, positive comovement does not exclusively result from using the particular functional form (6). In fact it can be easily recognized that positive comovement results from *any* utility function that is additively separable in its arguments. Additively separable functions represent a widely used class of utility functional forms. Moreover, non-separability of preferences has been rejected as an empirical explanation for the lack of international consumption risk sharing (see Lewis (1999)). Second, positive comovement does not only occur under complete asset markets, but, as it will be shown in the next sections, in *any* situation in which the exchange rate is exclusively determined by contemporaneous monetary policy stances as in (9). Note also that positive comovement resulting from (9) is independent from the production side. This suggests that even allowing for wage or price rigidities in a model with complete markets would not change the positive comovement of real exchange rate and aggregate consumption ratio. Hence it is not surprising that the sticky-price model of Chari, Kehoe, and McGrattan (2002) generates a high correlation under complete markets.

3 The incomplete asset market case

In the previous section it has been shown that positive comovement of real exchange rate and relative consumption under complete markets is related to process of the exchange rate determination. In this section I investigate what is the mechanism that determines the exchange rate in a situation in which domestic and foreign agents do no longer have a set of securities at their disposal that allow them to ensure against any state of the world. The standard way to model this form of market incompleteness is to assume that agents only have one-period unconditional bonds that can be traded internationally. I assume that there are two kinds of such bonds, one is denominated in domestic currency and the other one in foreign currency. Furthermore, I assume that the home and foreign country are both populated by a large number of identical, infinitely lived consumers indexed by $j \in [0,1]$ and $j^* \in [0,1]$, respectively. This implies that budget constraint under incomplete asset market takes the following form:

$$M_t(j) + B_{H,t+1}(j) + \varepsilon_t B_{F,t+1} \leqslant M_{t-1}(j) + (1+i_t)B_{H,t}(j) + (1+i_t^*)\varepsilon_t B_{F,t} \\ +w_t(j)L_t(j) + \Pi_t(j) - P_t C_t(j) - T_t(j) \qquad (10)$$

where $B_{H,t}(j)$ and $B_{F,t}(j)$ are domestic agent j holdings of the the international bonds denominated in domestic and foreign currency respectively, and i_t and i_t^* are their respective nominal yields paid at the beginning of period t but known at time $t-1$[4].

The incomplete market version of (3) and (6)) is:

$$\mathcal{U}_t(j) = E_t \sum_{t=\tau}^{\infty} \beta^{\tau-t} \left(\log C_\tau(j) + \chi_\tau \log \left(\frac{M_\tau(j)}{P_\tau)} \right) - \kappa_\tau L_\tau(j) \right) \qquad (11)$$

I assume that the full consumption basket $C_t(j)$ can be disaggregated into tradable and non-tradable goods consumption as follows:

$$C_t(j) = \frac{C_{T,t}(j)^\gamma C_{N,t}(j)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \qquad (12)$$

where $C_{T,t}(j)$ and $C_{N,t}(j)$ are the consumption of an aggregate of tradable goods and non-tradable goods, respectively. γ is the share of tradable good over total consumption expenditures. Furthermore, $C_{T,t}(j)$ can be disaggregated into consumption of a basket of home tradable goods $C_{H,t}(j)$ and consumption of a basket of home tradable goods and $C_{F,t}(j)$ as follows:

[4]The notation follows from (Obstfel and Rogoff 1996) and Corsetti and Dedola (2005). $M_t(j)$ denotes agent j's nominal balance accumulated during period t and carried over into period $t+1$, while $B_{H,t}(j)$ and $B_{F,t}(j)$ denote agent j's bonds accumulated during period t and carried over into period $t+1$.

$$C_{T,t}(j) = \frac{C_{H,t}(j)^{\alpha} C_{F,t}(j)^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{13}$$

where α is the share of home good over tradable consumption expenditures. Thus α is a natural index of home bias in consumption and id inversely related to the degree of country's openness. Finally, the aggregates $C_{H,t}(j)$, $C_{F,t}(j)$ and $C_{N,t}(j)$ are defined as:

$$C_{H,t}(j) = \left[\int_0^1 C_t(h,j)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}} \tag{14}$$

$$C_{F,t}(j) = \left[\int_0^1 C_t(f,j)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}} \tag{15}$$

$$C_{N,t}(j) = \left[\int_0^1 C_t(n,j)^{\frac{\theta-1}{\theta}} dn \right]^{\frac{\theta}{\theta-1}} \tag{16}$$

where $C_t(h,j)$,$C_t(h,j)$ and $C_t(n,j)$ denote consumption of the various varieties of domestic traded goods (indexed by h), of foreign traded goods (indexed by f) and of domestic non-traded goods (indexed by n), respectively. θ is the elasticity of substitution among varieties of these goods ($\theta > 1$).

Finally, the government budget constraint is:

$$\int_0^1 [M_t(j) - M_{t-1}(j)]dj = \int_0^1 T_t(j)dj \tag{17}$$

3.1 Household optimization

Following Obstfel and Rogoff (2000) and Corsetti and Dedola (2005), households are assumed to be monopolistic supplier of labor who set their wages one period in advance. Hence, the home agent j chooses consumption, real balances, bond holdings and leisure to maximize their lifetime utility (11) given (10) and the labor demand $L_t(j) = \left(\frac{w_t(j)}{W_t}\right)^{-\varpi} L_t$.

From the FOC with respect to $C_t(j)$, $M_t(j)$, $B_{H,t}(j)$, $B_{F,t}(j)$ and $L_t(j)$ it follows that (see Appendix 2 for details):

$$\frac{1}{P_t C_t(j)} = \beta(1+i_{t+1}) E_t \frac{1}{P_{t+1} C_{t+1}(j)} \tag{18}$$

$$\frac{(1+i_{t+1})}{(1+i^*_{t+1})} = E_t \left(\frac{\varepsilon_{t+1}}{P_{t+1} C_{t+1}(j)} \right) / E_t \left(\frac{\varepsilon_t}{P_{t+1} C_{t+1}(j)} \right) \tag{19}$$

$$\frac{M_t(j)}{P_t} = \chi \frac{1+i_{t+1}}{i_{t+1}} C_t(j) \tag{20}$$

$$w_t(j) = \kappa \varphi^W \frac{E_{t-1} l_t(j)}{E_{t-1} l_t(j)/\mu_t(j)} \tag{21}$$

where $\varphi^W = \frac{\varpi}{\varpi - 1}$. (18) is the Euler equation, (19) is the risk-adjusted uncovered interest rate parity, (20) is the money demand function, and (21) is the equation that determines optimal wage set one period in advance.

3.2 Exchange rate determination

In a symmetric equilibrium, with $\mu_t \equiv P_t C_t$ and $\mu^*_t \equiv P^*_t C^*_t$, combining (18) with its foreign counterpart yields:

$$\varepsilon_t = \frac{\mu_t}{\mu^*_t} \left(\frac{E_t \frac{1}{\mu_{t+1}}}{E_t \frac{1}{\varepsilon_{t+1} \mu^*_{t+1}}} \right) \tag{22}$$

Comparing the equilibrium exchange rate determination process under incomplete asset markets (22) with that under complete asset markets (9), a significant difference shows up: the exchange rate is no longer uniquely determined by the contemporaneous monetary stances. In fact it now also depends on the term $\frac{E_t \frac{1}{\mu_{t+1}}}{E_t \frac{1}{\varepsilon_{t+1} \mu^*_{t+1}}}$, i.e. on the expectations about the future, that agents build up relying on all information they have gathered until period t.

What is the implication on the comovement of real exchange rates and relative consumption? From (22), it follows that:

$$\underbrace{\frac{\varepsilon_t P^*_t}{P_t}}_{q_t} = \underbrace{\frac{\mu_t}{\mu^*_t} \frac{P^*_t}{P_t}}_{\frac{C_t}{C^*_t}} \frac{E_t \frac{1}{\mu_{t+1}}}{E_t \frac{1}{\varepsilon_{t+1} \mu^*_{t+1}}} \tag{23}$$

Equation (23) describes the relationship between relative consumption and real exchange rate in the incomplete market case. Compared to its complete market counterpart (9), there is now a expectation term that intervenes in determining the relationship. Comovement of these variables in the complete market case can thus be considered as a special case of the incomplete markets, namely the special case of (23) in which, $\frac{E_t \frac{1}{\mu_{t+1}}}{E_t \frac{1}{\varepsilon_{t+1}\mu^*_{t+1}}} = 1$.

To understand *how* the additional term change the relationship between relative consumption and real exchange rate, it is thus necessary to investigate the process of the exchange rate determination. The key point is that under incomplete asset markets the exchange rate is a forward-looking variable which is determined by the contemporaneous monetary stances and by agent's expectation about the future. This implies that the exchange rate, in addition to contemporaneous monetary policy innovations, also responds to all other innovations that take place in the home and foreign country, to the extent to which they affect agent's expectation. In the next section I explicitly model the production side. This allows to model innovations in the real side of the economy.

4 Production side

In each country it is assumed that that there are producers of intermediate goods, producers of tradable goods, producers of distribution services, and producers of non-traded goods. Each good comes in a continuum of varieties and is produced by monopolistic competitors. Different varieties are indexed by $i \in [0,1]$ for the intermediate good, $h \in [0,1]$ for the domestic traded good, $f \in [0,1]$ for the foreign traded good, $d \in [0,1]$ for the distribution services and $n \in [0,1]$ for the remaining non-traded goods. i^*,d^* and n^* are the indices for the foreign counterparts. The national and international flows of goods are represented in Figure A1.

4.1 Technology

Let $y_t(.)$ be the output of the producer of a representative variety of the various goods. Goods i, n d are produced using labor units provided by the domestic households. The productions functions are respectively:

$$y_t(i) = Z_{I,t} \left[\int_0^1 L_t(i,j)^{\frac{\varpi-1}{\varpi}} dj \right]^{\frac{\varpi}{\varpi-1}} \tag{24}$$

$$y_t(n) = Z_{N,t} \left[\int_0^1 L_t(n,j)^{\frac{\varpi-1}{\varpi}} dj \right]^{\frac{\varpi}{\varpi-1}} \tag{25}$$

$$y_t(d) = Z_{D,t} \left[\int_0^1 L_t(d,j)^{\frac{\varpi-1}{\varpi}} dj \right]^{\frac{\varpi}{\varpi-1}} \tag{26}$$

where $\left[\int_0^1 L_t(i,j)^{\frac{\varpi-1}{\varpi}} dj\right]^{\frac{\varpi}{\varpi-1}} \equiv L_t(i)$, $\left[\int_0^1 L_t(n,j)^{\frac{\varpi-1}{\varpi}} dj\right]^{\frac{\varpi}{\varpi-1}} \equiv L_t(n)$, $\left[\int_0^1 L_t(d,j)^{\frac{\varpi-1}{\varpi}} dj\right]^{\frac{\varpi}{\varpi-1}} \equiv L_t(d)$, are the aggregates of domestic labor inputs used by the producer of good i, n, d respectively . ϖ is the elasticity of substitution among labor inputs ($\varpi > 1$). $Z_{I,t}$,$Z_{N,t}$,$Z_{D,t}$ the sector-specific productivity parameters.

The tradable domestic (foreign) good is produced by assembling the domestic (foreign) intermediate goods. The production functions for the domestic and foreign tradable good producers are respectively:

$$y_t(h) = Z_{H,t} \left[\int_0^1 y_t(h,i)^{\frac{\nu-1}{\nu}} di \right]^{\frac{\nu}{\nu-1}} \tag{27}$$

$$y_t(f) = Z_{F,t} \left[\int_0^1 y_t(f,i^*)^{\frac{\nu-1}{\nu}} di^* \right]^{\frac{\nu}{\nu-1}} \tag{28}$$

where $\left[\int_0^1 y_t(h,i)^{\frac{\nu-1}{\nu}} di\right]^{\frac{\nu}{\nu-1}}$ $\left(\left[\int_0^1 y_t(f,i)^{\frac{\nu-1}{\nu}} di*\right]^{\frac{\nu}{\nu-1}}\right)$ is the aggregate of domestic (foreign) intermediate goods varieties i ($i*$) used to produce the home (foreign) tradable good. ν is a measure of the elasticity of substitution between the varieties i ($\nu > 1$). $Z_{H,t}$ and $Z_{F,t}$ are the productivity parameter specific to the home and foreign traded good sector respectively.

4.2 Distribution services

Distribution services enter the production chain at two stages. I distinguish between distribution services required to bring intermediate goods to the traded good producers (intermediate distribution) and distribution services required to bring tradable goods to the consumers (final distribution). I assume that bringing one unit of traded good to consumers requires η units of a basket $Y_{D,t}$ of distribution services. Similarly, bringing a unit of the intermediate good to the traded goods sector requires $\tilde{\eta}$ units of the same basket. Note that in this model, I don't take the usual assumption that makes distribution services equivalent to all other non-traded goods (see e.g. Erceg and Levin (1996), De Gregorio and Wolf (1994), Burnstein, Eichenbaum, and Rebelo (2005), Burnstein, Eichenbaum, and Rebelo (2006), Corsetti, Pesenti, and Leduc (2004), Corsetti and Dedola (2005)). In fact, as emphasized in section 1, the distribution sector has characteristics that make it differ from other non-traded good sectors.

The basket of distribution services is defined as:

$$Y_{D,t} \equiv \left[\int_0^1 y_t(d)^{\frac{\epsilon}{\epsilon-1}} dd \right]^{\frac{\epsilon-1}{\epsilon}} \tag{29}$$

where ϵ measures the elasticity of substitution among different varieties of distribution services ($\epsilon > 1$).

4.3 Price indexes

From (12)-(16) and (27)-(29) the following price indices can be derived:

$$P_t = \left(P_{H,t}^{\alpha} P_{F,t}^{1-\alpha} \right)^{\gamma} P_{N,t}^{1-\gamma} \tag{30}$$

$$P_{H,t} = \left[\int_0^1 p_t(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}} \tag{31}$$

$$P_{F,t} = \left[\int_0^1 p_t(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}} \tag{32}$$

$$P_{N,t} = \left[\int_0^1 p_t(n)^{1-\theta} dn \right]^{\frac{1}{1-\theta}} \tag{33}$$

$$P_{I,t} = \left[\int_0^1 p_t(i)^{1-\nu} di \right]^{\frac{1}{1-\nu}} \tag{34}$$

$$P_{D,t} = \left[\int_0^1 p_t(d)^{1-\epsilon} dd \right]^{\frac{1}{1-\epsilon}} \tag{35}$$

Finally, from (24)-(26) the wage rate index can be derived:

$$W_t = \left[\int_0^1 w_t(j)^{1-\varpi} dj \right]^{\frac{1}{1-\varpi}} \tag{36}$$

4.4 Firms optimization

Given production technologies, demand functions and the corresponding price indices it is possible to compute optimal prices charged by the firms (see Appendix 3 for details).

Producers of distribution services maximize their profits given the demand for their products. Given (29) the optimal prices for distribution services is:

$$p_t(d) = \varphi^D \left(\frac{W_t}{Z_{D,t}} \right) \tag{37}$$

where $\varphi^D = (\frac{\epsilon}{\epsilon-1})$

The optimization scheme applies to the producers of the all other non-tradeable goods. Given (16), the optimal price is:

$$p_t(n) = \varphi \left(\frac{W_t}{Z_{N,t}} \right) \tag{38}$$

where $\varphi = (\frac{\theta}{\theta-1})$

The producers of the intermediate goods set their price taking into account the demand for their product and the distribution services required to bring it to the next stage of the production chain. From (24), the demand for variety i can be derived:

$$y_t(i) = \left(\frac{p_t(i)}{P_{I,t}} \right)^{-\nu} Y_{I,t} \tag{39}$$

where, given the assumption about the complementarity of production and distribution, the relationship between the price including the distribution services $p_t(i)$ and the producer price $\overline{p_t}(i)$ (i.e. the price without distribution services) is:

$$p_t(i) = \overline{p_t}(i) + \tilde{\eta} P_{D,t} \tag{40}$$

The profit maximization implies that the producer of intermediate goods charge the following price:

$$\overline{p_t}(i) = \varphi^I \left(\frac{W_t}{Z_{I,t}} + \frac{\tilde{\eta}}{\nu} P_{D,t} \right) \tag{41}$$

where $\varphi^I = (\frac{\nu}{\nu-1})$

From (40), it follows that the price including distribution services, i.e. the price that producers of tradable goods h have to pay to buy one unit of good i, is:

$$p_t(i) = \varphi^I \left(\frac{W_t}{Z_{I,t}} + \tilde{\eta} P_{D,t} \right) \tag{42}$$

Producers of goods indexed by h sell their good in the home and in the foreign country. Given (14) and its foreign counterpart, it is possible to derive the demand functions for the home and foreign market, respectively:

$$C_t(h) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t}, \qquad C^*_t(h) = \left(\frac{p^*_t(h)}{P^*_{H,t}} \right)^{-\theta} C^*_{H,t}, \tag{43}$$

where, analogously to the previous case, there is the following relationship between the prices including of distribution services $p_t(h)$ and $p^*_t(h)$ and the producer price $\overline{p_t}(h)$,$\overline{p^*_t}(h)$:

$$p_t(h) = \overline{p_t}(h) + \eta P_{D,t}, \qquad p^*_t(h) = \overline{p^*_t}(h) + \eta P^*_{D,t} \tag{44}$$

The profit maximization yields the following optimal producer prices for the goods to be sold in the home and foreign market, respectively:

$$\overline{p_t}(h) = \varphi \left(\frac{P_{I,t}}{Z_{H,t}} + \frac{\eta}{\theta} P_{D,t} \right)$$
$$\overline{p_t^*}(h) = \varphi \left(\frac{1}{\varepsilon_t} \frac{P_{I,t}}{Z_{H,t}} + \frac{\eta}{\theta} P_{D,t}^* \right) \tag{45}$$

From (44) it follows that the price including of distribution services, i.e. the price that consumers have to pay for one unit of good h, is

$$p_t(h) = \varphi \left(\frac{P_{I,t}}{Z_{H,t}} + \eta P_{D,t} \right), p_t^*(h) = \varphi \left(\frac{1}{\varepsilon_t} \frac{P_{I,t}}{Z_{H,t}} + \eta P_{D,t}^* \right) \tag{46}$$

Note that in general $\overline{p_t}(h) \neq \varepsilon_t \overline{p_t^*}(h)$ and $p_t(h) \neq \varepsilon_t p_t^*(h)$. Pricing-to-market in the presence of distribution costs implies that the law of one price (LOOP) does not hold if productivity in the distribution sector and/ or wages are not equalized across countries, i.e. $Z_{D,t} \neq Z_{D,t}^*$ and/oder $W_t \neq \varepsilon_t W_t^*$.

Domestic and foreign price indices in the symmetric equilibrium are reported in Table 1.

Home country goods	Foreign country goods
$P_{I,t} = \varphi^I \left(\frac{W_t}{Z_{I,t}} + \tilde{\eta} P_{D,t} \right)$	$P_{I,t}^* = \varphi^I \left(\frac{W_t^*}{Z_{I,t}^*} + \tilde{\eta} P_{D,t}^* \right)$
$P_{D,t} = \varphi^D \left(\frac{W_t}{Z_{D,t}} \right)$	$P_{D,t}^* = \varphi^D \left(\frac{W_t^*}{Z_{D,t}^*} \right)$
$P_{N,t} = \varphi \left(\frac{W_t}{Z_{N,t}} \right)$	$P_{N,t}^* = \varphi \left(\frac{W_t^*}{Z_{N,t}^*} \right)$
$P_{H,t} = \varphi \left(\frac{P_{I,t}}{Z_{H,t}} + \eta P_{D,t} \right)$	$P_{F,t} = \varphi \left(\varepsilon_t \frac{P_{I,t}^*}{Z_{F,t}} + \eta P_{D,t} \right)$
$P_{H,t}^* = \varphi \left(\frac{1}{\varepsilon_t} \frac{P_{I,t}}{Z_{H,t}} + \eta P_{D,t}^* \right)$	$P_{F,t}^* = \varphi \left(\frac{P_{I,t}^*}{Z_{F,t}} + \eta P_{D,t}^* \right)$
$\overline{P}_{H,t}^* = \varphi \left(\frac{1}{\varepsilon_t} \frac{P_{I,t}}{Z_{H,t}} + \frac{\eta}{\theta} P_{D,t}^* \right)$	$\overline{P}_{F,t} = \varphi \left(\varepsilon_t \frac{P_{I,t}^*}{Z_{F,t}} + \frac{\eta}{\theta} P_{D,t} \right)$
$W_t = \kappa \varphi^W \frac{E_{t-1} l_t}{E_{t-1} l_t / \mu_t}$	$W_t^* = \kappa \varphi^W \frac{E_{t-1} l_t^*}{E_{t-1} l_t^* / \mu_t^*}$

Table 1: *Equilibrium Prices*

5 Conditional comovement analysis

In section 3 it has been shown that when asset markets are incomplete the exchange rate is no longer restricted to respond only to innovations in monetary policy stances, but it could potentially respond to any innovations that induce a revision of agent's t expectation about period $t+1$.

The strategy I take to investigate comovement of real exchange rate and relative consumption consists in decomposing such comovement into its components, in other words in analyzing the changes induced in these variables by each one of the various innovation sources, namely the monetary policy shocks and the technology shocks modeled in section 5. Making comovement conditional on the shocks and analyzing the interaction of the various conditional comovements will turn out to be a convenient strategy to understand how comovement differ under complete or incomplete asset markets. The conditional comovement are computed by log-linearizing consumption ratio $\frac{C_t}{C_t^*}$ and the real exchange rate $q_t = \frac{\varepsilon_t P_t^*}{P_t}$ around the following symmetric equilibrium:

$$\begin{gathered} Z_{H,t} = Z_{F,t} = Z_T, Z_{I,t} = Z_{I,t}^* = Z_I, Z_{D,t} = Z_{D,t}^* = Z_D \\ \mu_t = \mu_t^* = \mu \\ W_t = W_t^* = W = \varphi^L \kappa \mu \end{gathered} \tag{47}$$

Using standard notation, $\hat{X}_t$ gives the percentage deviation from its symmetric equilibrium value (denoted with X), i.e. $\hat{X}_t \cong \frac{X_t - X}{X}$.

Suppose that the economy is hit by a domestic monetary policy only (i.e. $\widehat{\mu_t^*} = \widehat{Z_{H,t}} = \widehat{Z_{F,t}^*} = ... = \widehat{Z_{N,t}} = \widehat{Z_{N,t}^*} = 0$ but $\widehat{\mu_t} \neq 0$) and the relationship between changes in the consumption ratio and the real exchange rate simplifies to $\widehat{\frac{C_t}{C_t^*}} = \xi \widehat{q_t}$. The coefficient ξ can be conveniently used to measure the comovement of these variables. The sign of ξ gives the sign of comovement, while the size of ξ tells the relationship between the magnitude of the changes in the two variables. For instance, $\xi = 0.5$ would mean that the percentage change in the consumption ratio is half of that in real exchange rate. I denote ξ *conditional comovement coefficient* to emphasize that it depends on a particular shock- in this example $\xi = \xi(\widehat{\mu_t})$.

In this section I work out not only the conditional coefficients for comovement of real exchange rate and relative consumption, but also for two other key comovement patterns: comovement of real and nominal exchange rates, and comovement of term-of-trade $\tau_t = \frac{\overline{P}_{F,t}}{\varepsilon_t \overline{P}^*_{H,t}}$ and nominal exchange rate. The reason is that these two comovements have also been investigated empirically, and thus it is possible to set up comparisons between these three sort of comovements, which allow a better understanding of their peculiarities.

Let's define:

$$\delta \equiv \frac{\eta P_D}{P_T} = \frac{\eta \varphi^D \frac{W}{Z_D}}{\varphi \left[\frac{P_I}{Z_T} + \eta \varphi^D \frac{W}{Z_D} \right]} \tag{48}$$

and

$$\tilde{\delta} \equiv \frac{\tilde{\eta} P_D}{P_I} = \frac{\tilde{\eta} \varphi^D \frac{W}{Z_D}}{\varphi^I \left[\frac{W}{Z_I} + \tilde{\eta} \varphi^D \frac{W}{Z_D} \right]} \tag{49}$$

as the share of distribution costs in the price of the tradeable and intermediate good, respectively. Hence, δ and $\tilde{\delta}$ give the so-called "distribution margins", (see Burnstein, Eichenbaum, and Rebelo (2006))

It is also convenient to define:

$$\omega \equiv (1-\alpha)\gamma(1-\delta\varphi) = \frac{\partial \left(\widehat{P}_t \right)}{\partial \widehat{\varepsilon}_t} \tag{50}$$

ω gives the percentage change in the domestic CPI due to a one percent change of the exchange rate. In other words, ω gives the degree of exchange rate pass-through to the consumer price index. The term $(1 - \delta\varphi)$ gives the percentage increase in the final price of the imported goods due to a 1 percent depreciation of the exchange rate.

Using (48)-(50) it is possible to derive the various conditional comovement coefficients. The conditional comovement coefficients of consumption ratio and real exchange rate are reported in Table A1. The conditional comovement coefficients of real and nominal exchange rates are reported in Table A2. Finally, the conditional comovement coefficients of term-of-trade and nominal exchange rates are reported in Table A3.

Note that symmetry across countries implies that it does not play a role in what country shocks take place; for instance comovements induced by $\widehat{Z_{I,t}}$ are the same as those induced by $\widehat{Z_{I*,t}}$. From Tables A1-A3, it can be seen that the various conditional comovement coefficients are not equal across shocks. In particular, the coefficients depend on some key factors: (i) the importance of tradable/non-tradable goods consumption γ, (ii) the home bias degree α, (iii) the elasticity of substitution between traded goods θ, (iv) the distribution margins δ and $\tilde{\delta}$ and, last but not least, (v) the exchange rate channel, i.e. the mechanism through which the shock's impact on the exchange rate.

As it has been shown in section 3, the functioning of the exchange rate channel changes depending on whether asset markets are complete or not. In the next section I analyze in detail the comovements resulting from alternative asset market specifications.

6 Conditional comovement analysis under complete and incomplete markets

First of all, I consider the situation in which asset markets are complete. Then I focus on the market incompleteness case. In addition to the specification from section 2, in which (uncontingent) international bonds are available (hereinafter, *bond economy*), I will also consider the extreme form of market incompleteness, namely the one in which no assets at all are available (hereinafter, *financial autarky*). Using a calibration of the key parameters, I will show how comovements differ qualitatively and quantitatively.

6.1 Conditional comovements under complete markets

How do comovements look like under complete markets when they are made conditional on the various monetary and technology shocks? The functioning of the exchange rate channel under complete markets is straightforward. In fact log-linearizing (9) yields:

$$\frac{\partial \widehat{\varepsilon}_t}{\partial \widehat{\mu}_t} = 1$$

$$\frac{\partial \widehat{\varepsilon}_t}{\partial \widehat{Z_{H,t}}} = \frac{\partial \widehat{\varepsilon}_t}{\partial \widehat{Z_{I,t}}} = \frac{\partial \widehat{\varepsilon}_t}{\partial \widehat{Z_{D,t}}} = \frac{\partial \widehat{\varepsilon}_t}{\partial \widehat{Z_{N,t}}} = 0$$

Changes in the exchange rate are *only* due to monetary policy shocks, namely a one percent change in the monetary stances is followed by a one percent depreciation of the exchange rate. Note that symmetry implies that responses to foreign shocks are identical, just with opposite sign ($\frac{\partial \widehat{\epsilon}_t}{\partial \widehat{\mu_t^*}} = -\frac{\partial \widehat{\epsilon}_t}{\partial \widehat{\mu}_t}$).

Using these information, it is easy to see from table 2 that all shocks induce a perfect (= 1) positive comovement between consumption ratio and real exchange rate.

The picture is quite different for the comovement of real and nominal exchange rates and for the comovement of term-of-trade and nominal exchange rates. In fact, in these cases comovement take place only following monetary policy shocks:

$$\widehat{q}_t = [1 - \omega]\widehat{\varepsilon}_t$$

$$\widehat{\tau}_t = \left[1 - 2\left(\frac{\delta(\varphi - 1)}{1 - \delta}\right)\right]\widehat{\varepsilon}_t$$

Comovement is however zero after all technology shocks, because changes in the real exchange rate and the term-of-trade are not accompanied by any changes in the nominal exchange rate.

6.2 Conditional comovements under financial autarky

Under the extreme case of market incompleteness, agents cannot borrow or lend internationally and hence it must be that the trade account is perfectly balanced. This implies that the exchange rate can be pinned down as follows:

$$\varepsilon_t = \frac{\overline{P}_{F,t} C_{F,t}}{\overline{P}^*_{H,t} C^*_{H,t}} \tag{51}$$

Using the fact that in equilibrium $P_{F,t}C_{F,t} = \alpha\gamma P_t C_t = \alpha\gamma\mu_t$ and $P^*_{H,t}C^*_{H,t} = \alpha\gamma P^*_t C^*_t = \alpha\gamma\mu^*_t$, this can be rewritten as:

$$\varepsilon_t = \frac{\mu_t}{\mu^*_t} \frac{\frac{\overline{P}_{F,t}}{P_{F,t}}}{\frac{\overline{P}^*_{H,t}}{P^*_{H,t}}} \tag{52}$$

Log-linearizing (52) and thereby taking into account optimal prices reported in table 1 yields:

$$\frac{\partial \widehat{\varepsilon_t}}{\partial \widehat{\mu_t}} = \frac{1}{1 - 2\delta\left(\frac{1-\varphi\delta}{1-\delta}\right)}$$

$$\frac{\partial \widehat{\varepsilon_t}}{\partial \widehat{Z_{H,t}}} = \frac{\delta\left(\frac{1-\varphi\delta}{1-\delta}\right)}{1 - 2\delta\left(\frac{1-\varphi\delta}{1-\delta}\right)}$$

$$\frac{\partial \widehat{\varepsilon_t}}{\partial \widehat{Z_{I,t}}} = \frac{(1-\tilde{\delta})\delta\left(\frac{1-\varphi\delta}{1-\delta}\right)}{1 - 2\delta\left(\frac{1-\varphi\delta}{1-\delta}\right)}$$

$$\frac{\partial \widehat{\varepsilon_t}}{\partial \widehat{Z_{D,t}}} = \frac{(1+\tilde{\delta})\delta\left(\frac{1-\varphi\delta}{1-\delta}\right)}{1 - 2\delta\left(\frac{1-\varphi\delta}{1-\delta}\right)}$$

$$\frac{\partial \widehat{\varepsilon_t}}{\partial \widehat{Z_{N,t}}} = 0$$

Before moving to the results, it is worth to compare how the exchange rate channel works in the two alternative asset market structures. Unlike complete markets, in which only monetary policy shocks can affect the exchange rate in a one-to-one relationship, exchange rates under financial autarky can also be affected by all technology shocks, with the only exception of productivity shocks in the non-traded sectors. Furthermore, the impacts now depend on the distribution margins δ and $\tilde{\delta}$, as well as on θ ($\varphi = \frac{\theta}{\theta-1}$). In particular the sign of the responses is given by the interaction of δ and θ in the denominator. In the extreme case of perfect substitutability among the varieties of the traded goods ($\theta \to \infty$), all responses are positive if δ is larger than 0.5. However, this critical value increases if the degree of substitutability is not perfect. For instance, $\theta = 10$ implies that the critical value raises to 0.6, i.e. to a number that is higher than the estimates of distribution literature from the empirical literature (See Burnstein, Neves, and Rebelo (2003) and Anderson and VanWincoop (2004)).

The importance of distribution services at the final level becomes particularly clear if ones considers the extreme case in which no distribution services are required to distribute tradable goods to the final consumers. In fact it is easy to recognize that $\delta = 0$ would imply that the exchange rate channel under financial autarky would become identical to the one of complete markets (see equation (52), which in turn would imply that under financial autarky there would be perfect positive comovement of consumption ratio and real exchange rate as in under complete markets!

Making use of the the log-linearization of (52) it is possible to rewrite the conditional coefficients from Tables A1-A3 for the special case of financial autarky. The results are reported in Tables A4-A6. Comparing these tables it is possible to make three remarks. First, technology shocks taking place in the production of intermediate and tradable goods induce the same comovement of the variables. The reason for this is that such shocks affect the production of the same tradable good just at different stages.

Second, all shocks are able to generate comovement between the variables. The only exception is given by technology shocks in the non-traded sectors, following which there no comovement of real and nominal exchange rates and no comovement of term-of-trade and nominal exchange rates, simply because the nominal exchange rate does not move at all.

Last but not least, the distribution margin at the intermediate level $\tilde{\delta}$ influences the exchange rate channel mechanism, but its effect becomes negligible in the context of comovements of the variables. Numerical experiments show in fact that the presence of distribution services at the intermediate level does not significantly affects conditional comovement patterns. For this reason, from now on I will only consider distribution services at the final level by assuming $\tilde{\eta} = 0$.

6.3 Conditional comovement in the bond economy

After the special case, I consider the standard case of market incompleteness, in which trade in international bonds is allowed and agents face flow budget constraint (10). Obviously, in this situation the trade account must not necessarily be balanced. This implies that it is no longer possible to derive analytically the comovement coefficients as in the previous subsection, and that numerical experiments are required (see Appendix 4 for details).

6.4 Calibration and simulation results

As it has been shown previously, the conditional comovement coefficients are essentially determined by γ, α, θ and δ. In this section I calibrate these parameters to match a set of stylized facts emerging from empirical literature on industrialized countries. The idea is to capture is to capture the dynamic interactions between two similar "average" industrialized countries as in Stockman and Tesar (1995). First of all, δ is set to 0.5 according to the available estimates of distribution margins (see Burnstein, Neves, and Rebelo (2003) and Anderson and VanWincoop (2004)). The degree of substitutability is set to 10 such that together with the distribution size it gives an exchange rate pass-through to import prices net of distribution costs close to 0.8, thus in line with the available estimates (see Cavaliere (2005) and Campa and Goldberg (2004)). γ is set to 0.5 such that tradable and non-tradable have the same weight in consumption. Finally, α is set to 0.5, such that together with the other parameters it gives an exchange rate pass-through to consumer prices which is close to 0.2, in line with the available estimate (see Cavaliere (2005), Gagnon and Ihrig (2004) and Bailliu and Fujii (2004)). The value of the conditional comovement coefficients obtained with this benchmark calibration are reported in Table 2:

	$\widehat{\frac{C_t}{C_t^*}} = [...]\widehat{q}_t$			$\widehat{q}_t = [...]\widehat{\varepsilon}_t$			$\widehat{\tau}_t = [...]\widehat{\varepsilon}_t$		
Shock	*Complete Markets*	*Financial Autarky*	*Bond Economy*	*Complete Markets*	*Financial Autarky*	*Bond Economy*	*Complete Markets*	*Financial Autarky*	*Bond Economy*
$\widehat{\mu}_t$	1	-0.15	0.29	0.78	0.78	0.78	0.76	0.76	0.76
$\widehat{Z_{H,t}}$	1	-0.29	-0.27	-	0.77	0.75	-	0.89	0.91
$\widehat{Z_{D,t}}$	1	-0.18	-0.16	-	0.84	0.82	-	0.75	0.71
$\widehat{Z_{N,t}}$	1	1	0.58	-	-	0.79	-	-	0.76

Table 2: *Conditional comovement coefficients under alternative asset market specifications*

These values can be used to assess how the various model specifications fit the data. The evaluation relies on the following crucial point: *unconditional* comovement is the result of the interaction of the various *conditional* comovements. If conditional comovements share the same characteristics, i.e. they are quantitatively and qualitatively quite similar, then it is possible to derive implications for the unconditional comovement. Conditional comovement coefficients are characterized by two things: sign and magnitude. A positive conditional coefficient implies a positive conditional correlation, and viceversa. A coefficient larger than one in modulus would imply that the left-hand variable should be more volatile than the right-hand variable.

I start with the cases in which asset market are incomplete. The bond economy and the special case of financial autarky only differ with respect to two points[5]. The first one concerns comovement of real exchange rate and relative consumption, which following monetary policy shocks is positive in the bond economy and negative under financial autarky. This is due to the fact that financial autarky requires a larger depreciation of the exchange rate after a monetary policy shock compared to the bond economy. The second difference concerns comovement of the other variables following shock in the nontradable sector. Here comovement is zero under financial autarky and positive in the bond economy, simply because in the latter case the exchange rate can respond to such shocks. Concerning this aspect, it should however emphasized that the literature tends to consider such productivity changes less important in comparison to the others[6].

The comovements that take place under incomplete markets are in line with the empirical evidence from several key viewpoints:

- The coefficients of conditional comovement between real exchange rate and consump-

[5] This is not surprising as several contributions show in fact that the specification of market incompleteness does not have a significant incidence on the international transmission mechanism. See for instance Bodenstein (2005) and Corsetti and Dedola (2005).

[6] A classical example is the Balassa-Samuelson hypothesis which conjectures that productivity increases in the tradable good sector tend to be higher than those in the nontradable sectors. Evidence about the low productivity in the nontradable sector can be found for instance in De Gregorio and Wolf (1994), Takatoshi, Isard, and Symansky (1997) and Aebersold and Brunetti (1998).

tion ratio are all smaller than one in modulus and do not have the same sign. This implies that: (*i*) unconditional correlations should be low, as documented in Backus and Smith (1993), Kollman (1995), Ravn (2001) and Corsetti, Pesenti, and Leduc (2004); (*ii*) consumption ratios should be less volatile than real exchange rates, as documented in Backus and Smith (1993).

- Productivity shocks in the tradable goods sector generate a negative comovement of consumption ratio and real exchange rate. This implies that the correlation conditional on such shocks should be negative, as documented in Corsetti, Pesenti, and Leduc (2004)

- The coefficients of conditional comovement between real and nominal exchange rates and between term-of-trade and nominal exchange rates are all positive and somewhat smaller than one. This implies that: (*i*) unconditional correlations between these variables should be strong[7], as documented in Chari, Kehoe, and McGrattan (2002), Mussa (1986) and Obstfel and Rogoff (2000); (*ii*) real exchange rates and term-of-trades should be somewhat less volatile then nominal exchange rates, as documented in Chari, Kehoe, and McGrattan (2002) and Obstfel and Rogoff (2000))

- Monetary policy shocks generate a positive comovement of real and nominal exchange rates. This implies that the correlation conditional on such shocks should be positive, as documented in Eichenbaum and Evans (1995).

Note that the picture looks quite different when complete asset markets are assumed. In fact, all shocks induce a perfect positive comovement of consumption ratio and real exchange rate. For the other two kind of comovements, real exchange rate and term-of-trade move similarly to nominal exchange rate only after monetary policy shocks; after all the other shocks comovement is zero as the the nominal exchange rate does not move. Hence complete

[7]Obstfel and Rogoff (2000) consider the positive comovement between the term-of-trade and the nominal exchange rate as key requirement to macroeconomic models. Relying on this requirement they reject models that feature local-currency-pricing (LCP) of imports.

markets would imply: (*i*) a perfect positive correlation of consumption ratio and and real exchange rate (as shown in section 2), (*ii*) a much lower correlation of the other variables, (*iii*) a volatility of consumption ratio similar to that of real exchange rate. However, these implications clearly contradict the empirical evidence mentioned above.

6.5 Sensitivity analysis

Results from the previous subsection indicate that comovements under incomplete asset markets seem thus to match empirical evidence more closely than those generated under complete markets. The question is how sensitive this conclusion is to the particular calibration that has been chosen to derive the results. Figures 1-3 plot the conditional comovements generated by a broad range for α and δ under financial autarky. Note that these parameters could in principle range from 0 to 1, however, to exclude implausible values of distribution margins and home bias I assume $\delta \in [0; 0.7]$ and $\alpha \in [0.3; 1]$ respectively. Figure 1 shows the conditional comovement of consumption ratio and real exchange rate, Figure 2 shows the conditional comovement between real and nominal exchange rates, Figure 3 shows the conditional comovement between term-of-trade and nominal exchange rate. In these plots, γ and θ take the benchmark values.

From Figures 1-3 it follows that under financial autarky the coefficients of conditional comovements are always less than one in modulus. As expected, the coefficients in Figure 2-3 are close to one. Furthermore, the coefficients in Figures 2-3 are always positive. Since numerical experiments confirm that these conclusions also hold in the bond economy, it is possible to formulate two propositions that are robust to the calibration of α and δ:

Proposition 1. *Under incomplete markets the model implies that consumption ratios are less volatile than real exchange rates. Furthermore, the model implies that the volatility of nominal exchange rates is very close to the one of real exchange rate and of term-of-trade.*

Proposition 2. *Under incomplete markets the model implies a positive correlation between nominal and real exchange rates and between term-of-trades and nominal exchange rates.*

What about the correlation between consumption ratios and real exchange rates? Figure 1 shows that under financial autarky the conditional comovement coefficients could change sign depending on the calibration of α and δ. Figure 4 reports the sign of these coefficients as a function of these parameters, abstracting from technology shocks in the non-traded sectors. The space generated by α and δ could theoretically be divided in three regions: a first region in which conditional comovements would be all positive, a second region in which conditional comovements would be all negative and, finally, a third region in which conditional comovement would be positive or negative depending on the shock. In the case of the first two regions, parameter combinations would imply a strong positive, respectively negative correlation. Parameter combination in the third region would generate instead comovements that go in opposite directions, which implies weak correlations with positive or negative sign depending on what shocks dominate.

Under plausible calibrations, i.e. δ and α *around* 0.5, financial autarky implies that we should be in the region of 'strong negative' correlation of consumption ratio and real exchange rate. The strength of this correlations is however likely to decrease when technology shocks in the non-traded sectors are considered. In fact, these shocks differ from the others as they generate a positive comovement. Switching from the financial autarky special case to the bond economy case, basically reduces further the correlation, because in the bond economy monetary policy shocks also generate a positive comovement. Hence it is possible to formulate a proposition which is robust to any plausible calibration of the parameters[8]:

Proposition 3. *Under complete markets the model implies a weak correlation of real exchange rate and consumption ratio.*

Summarizing, all this suggests that results from section 6.4. do not exclusively depend on the particular benchmark calibration, but they are instead robust to any plausible calibration of the parameters.

[8]Propositions 1-3 are also robust to the choice of γ and θ.

7 The role of distribution services and openness

The robustness analysis has emphasized the importance of distribution services and openness/home bias in consumption. The objective of this final section is to investigate how these elements interact in determining the comovement patterns documented in the previous section. First, I discuss the implication of considering distribution services, then I show what is the role played by the home bias in consumption.

7.1 Distribution services

In the model I've presented in section 4, distribution services are required at two stages of the production-consumption chain for tradeable goods. Distribution services at the intermediate level intervene in affecting the transmission of productivity shocks occurring in the intermediate good and distribution sectors, but their impact becomes negligible in the context of the comovement patterns. On the contrary, distribution services at the final level play a crucial role in affecting comovement of the macroeconomic variables. There are two aspects to emphasize. The first one is that such distribution services determine the sign of the exchange rate's response to the shocks. As it has been shown in section ??, estimates for the distribution margin at the final level suggest that when markets are incomplete the exchange rate depreciates after any domestic shocks, i.e. unconditionally. Since comovement of real and nominal exchange rates has been found to be unconditionally positive, it must follow that also the real exchange rate should depreciate after any domestic shock. This conclusion is confirmed by table A1. In fact, it is easy to recognize that the sign of the real exchange rate's response to a given shock coincides with the sign of the denominator of the corresponding conditional comovement coefficient. Being the degree of exchange rate pass-through to the consumer price index less than one (i.e. $\omega < 1$), all denominators must be positive and the real exchange rate's response is hence unconditionally positive.

The second aspect does not concern the sign, but the magnitudes of the response of nominal exchange rate. As emphasized in Corsetti, Pesenti, and Leduc (2004), distribution services decrease the price elasticity of imports. In fact, (13) implies that the elasticity of imports $C_{F,t}$ to the price $\overline{P}_{F,t}$ is equal to $1 - \frac{\eta P_{N,t}}{P_{F,t}} \equiv 1 - \delta_{F,t}$. Any shock that induces a decrease of imported good price will induce a proportionally smaller increase in the quantity of the imported good. In order to avoid a trade account deficit, the exchange rate needs then to depreciate. The larger is the distribution margin, the more over-proportional will be the required adjustment in the exchange rate.

7.2 Openness

Since the real exchange rate depreciates after any domestic shocks, it follows that the sign of comovement of consumption ratio and real exchange rate will be determined by the sign of the response of the consumption ratio, i.e by the numerators of the coefficients reported in Table A1. But the exchange rate positive response also implies that every shock has both a direct and an indirect effect on the consumption ratio. The direct effect is positive while the indirect is negative. To see that, let's consider a domestic monetary monetary policy shock, i.e. an increase in μ_t. Such shock has a direct positive effect because it gives domestic consumers more money to perform consumption expenditures. The shock has, however, also a negative indirect effect because it induces a depreciation of the exchange rate, which is then transmitted via the exchange rate pass-through channel (i.e. through ω) to an increase in the domestic CPI P_t and to a decrease in the foreign price P_t^*. For technology shocks, the positive effect is the usual one that induces a decrease of domestic price level relative to foreign price level due to productivity increase, while the negative indirect effect is, once again, due to the depreciation and the exchange rate pass-through mechanism.

Concerning the indirect effect, two points have to be made. The first one is that the depreciation following domestic shocks is not negligible. In fact, as emphasized in the previous subsection, the presence of distribution services at the final level implies that a domestic shock induced a over-proportional depreciation of the exchange rate. The second one is that the transmission of the depreciation to the domestic and foreign CPI depends on the degree of home bias in consumption. In fact, (50) implies that the higher is α, the lower is the degree of exchange rate pass-through to consumer prices ω.

The more open is the country (or, equivalently, the less consumers are biased towards consumption of tradeable goods produced in their own country), the larger is the negative effect on the consumption ratio. Note that in the case of productivity shocks that takes place in the tradable sectors (Z_H or Z_I), a low the home bias degree not only magnifies the negative effect, but it also reduces the positive effect. The reason is that an increase in domestic tradeable goods productivity induce a reduction of both the domestic (because of lower $P_{H,t}$) and the foreign price level (because of lower $P^*_{H,t}$). The lower is α the more the second impact dominates over the first.

7.3 Implications for the Backus-Smith correlation

If consumers would be fully biased towards their own tradeable goods, comovement of real exchange rate and consumption ratio would be positive for any given shock, and the model would predict a high degree of risk-sharing even under incomplete markets. However, given the large liberalization of trade that has taken place after the second world war, it is clear that home bias is far from being perfect and hence any exchange rate adjustment will be pass-through to the aggregate prices. Since the presence of distribution services magnifies such adjustments, it follows that the depreciation-induced indirect effect discussed in the previous subsection is not negligible. For any given value of α less than one, we thus have that the direct and indirect effects are at work and the result of their interaction will be positive or negative depending on the shock. Monetary policy and non-traded sector productivity shock are likely to generate positive comovement, while traded and distribution sector productivity shocks are likely to generate negative comovement. The following proposition can thus be stated:

Proposition 4. *Under incomplete markets the model implies that the sign of conditional comovement of real exchange rate and relative consumption coincides with the sign of the response of consumption ratio to the shocks. This sign is in turn determined by the interaction of a direct, positive and a indirect, negative effect. For plausible degrees of country's openness, the result of this interaction gives a comovement that is negative or positive depending* *on what shock it has been generated.*

When adjustments of consumption ratio and real exchange rate depends on the shock that generates them, then the sign of the correlation observed in the data is the result of the adjustments following all the shocks that have taken place in the economy. This sign would be negative if technology shocks in the traded sector and in the distribution sectors would be more frequent and stronger than monetary policy shocks and technology shocks in the nontradable sector, and viceversa. In other words the correlation sign would be determined by what shocks dominate the others. Given the emphasis about the importance of innovations that have taken place in the traded and distribution sectors in the last decades, it is not surprising that correlation were mostly negative.

8 Conclusion

In this contribution I present a two-country model that features distribution services at different stages of the production-consumption chain. I show that when asset markets are incomplete the model can generate comovement of variables, which are consistent with empirical regularities regarding volatility and correlation like it is hardly the case under complete markets. In particular I show that the model is consistent with the findings about low correlation of consumption ratios versus high correlations of real and nominal exchange rates and term-of-trade and nominal exchange rate. To illustrate that I decompose comovement of these variables into the comovements conditional on the various shocks that take place in the economy. Comovement of the Backus-Smith variables differ from the other comovements in the sense that it can be either positive or negative depending on the what shock it has been generated. I show that this mechanism is due to *(i)* the presence of distribution services at the final level of the production-consumption chain and *(ii)* the fact that countries are open. In fact, these elements are responsible for a depreciation-induced re-adjustment of prices and consumption quantities that runs counter the adjustments which are consistent with optimal risk-sharing patterns. The result of the interaction of these effects depends on the kind of shock that takes place. Monetary policy and non-traded sector productivity shock are more likely to generate positive comovement or real exchange rate and consumption ratio, while traded and distribution sector productivity shocks are more likely to induce adjustments that are negatively related and thus not consistent with optimal risk-sharing. This suggests that the fact that Backus-Smith correlations in the data are low might result from the combinations of these shock-specific positive and negative comovements in these variables. The fact that such correlations tend to be mostly (weak and) negative is consistent with the emphasis about larger productivity increases in the traded and distribution sectors.

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