Negative Indemnities in Health Insurance

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ABSTRACT

In the literature on optimal indemnity schedules, indemnities are usually restricted to be non-negative. Keeler (1974) and Gollier (1987) show that this constraint might well bind: insured could get higher expected utility if insurance contracts allowed payments from the insured to the insurer for some (small) losses. Considering these results and some extensions provided in this paper, it turns out that there should be a case for negative indemnities in health insurance. To find some reasons why negative indemnities are not observed in practice, the paper's final section introduces asymmetric information and shows that moral-hazard may change the results derived for the case of symmetric information considerably.

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1. Introduction

An insurance contract is usually defined as an agreement between an insurer and an insured in which the insurer promises to pay non-negative indemnities depending in the amount of loss suffered by the insured, for which in return the insured pays a fixed premium irrespective of the state of the world. To restrict indemnities to be non-negative appears to be most sensible, since it prevents risk averse insured to become insurers themselves. However, as Keeler (1974) and Gollier (1987) point out, this restriction can be binding for some loss distributions. In other words, under certain circumstances, the (risk averse) insured can get higher expected utility if they are allowed to sign contracts that provide for payments from the insured to the insurer for some losses. This result holds even for a risk neutral insurer.

While Keeler presents the basic idea in a strikingly intuitive way, Gollier is mathematically more specific and obtains the following results for insurance contracts that do not impose a non-negativity constraint on indemnities under symmetric information:

1. As in insurance contracts of the usual deductible type (Arrow, 1971), optimal contracts show a (non-negative) loss x_+ that acts as a deductible. For all losses above the deductible marginal indemnity is 1, thus providing full marginal insurance, and indemnity amounts to $I(x) = x - x_+$ for $x > x_+$.

2. Optimal contracts might contain a (non-negative) loss x_- with $x_- \le x_+$. For all losses between zero and x_- indemnity is negative and marginal indemnity equals 1. Consequently, indemnity payments for $x < x_-$ are given by $I(x) = x - x_-$.

3. For all losses between x_- and x_+ indemnities are zero

4. For the lower bound, x_-, it is true that $x_- \le F^{-1}(1/2)$, with $F(x)$ representing the cumulative distribution function of losses x. The practical consequence of this result is that the non-negativity constraint is never

binding if the probability of suffering a loss is less than $1/2$, which is obviously the case for many insured incidents.

This paper starts by generalizing Gollier's results, which depend on a special choice of the insurer's cost function. It is shown that marginal indemnity will generally be smaller than 1 and that the non-negativity constraint on indemnities may more likely be binding if we allow for more general cost functions.

Given Gollier's and our results, the non-negativity constraint should frequently be binding in health insurance. However, negative indemnities do not play any role in this branch of insurance. This could indicate that insurance schedules featuring negative indemnities may be plausible under symmetric information in the insurance market but may fail to hold if moral hazard is present. The last section of this paper therefore investigates the applicability of insurance contracts featuring negative indemnities in health insurance under moral hazard.

The basic properties of an indemnity schedule which allows for negative indemnities are introduced in section 2. Section 3 investigates if the non-negativity constraint usually imposed on indemnity schedules is actually binding. Finally, section 4 introduces moral-hazard considerations. It will be shown that indemnity schedules featuring negative indemnities are appropriate in the case of self-insurance but may not be optimal if we are concerned about pure self-protection.

2. The model

While Gollier gets his results by applying calculus of variation, this paper will (in line with Raviv (1979)) employ optimal control.

2.1 Assumptions

Let risk averse individuals have utility function $U(A)$, $U'(A) > 0$, $U''(A) < 0$, with A representing their net wealth. The risk neutral insurer is supposed to recover costs but to make zero expected profit. Premiums (P) therefore are equal to expected indemnity plus administrative costs C, which also emerge when indemnities are negative:

(1) $$P = \int_0^L \left(I(x) + C\left(\left| I(x) \right| \right) \right) f(x) dx ,$$

with $f(x)$ representing the density function of losses and L the maximal possible loss. Insured cannot affect the loss distribution. $I(x)$ represents the (positive or negative) indemnity. Marginal costs depend on absolute indemnities and are to be positive and increasing: $C'\left(\left| I(x) \right| \right) \geq 0$ and $C''\left(\left| I(x) \right| \right) \geq 0$.

In contrast, Gollier assumes costs to amount to $C\left(E(\left| I(x) \right|) \right)$. Consequently, in his model the premium reads as

(2) $$P_{Gol} = \int_0^L I(x) f(x) dx + C\left(\int_0^L \left| I(x) \right| f(x) dx \right).$$

While Gollier's premium function is adequate if the insurer has to bear costs which depend exclusively on the average amount of losses (e.g. personnel, IT), premium function (1) is more adequate if the insurer's costs also depend on the actual amount of losses, which seems to be more realistic since higher losses might call for more investigations, expert reports and so on. Furthermore, observe that (1) is compatible with Gollier's premium function (2) if marginal costs in (1) are constant $\left(C'' = 0 \right)$. Therefore (2) can be regarded as a special case of (1).

Let w denote individuals' exogenous wealth. The optimal insurance contract is found by maximizing insured's expected utility

(3) $$\int_0^L U\left(w - P - x + I(x) \right) f(x) dx$$

w.r.t. $I(x)$ subject to (1). The common constraint $I(x) \leq x$ is disregarded for two reasons: First, this constraint does not make much sense in a model as long as we do not allow for informational asymmetries, specifically moral hazard. Second, it will turn out that this restriction is not binding anyway if costs are strictly convex.

2.2 Optimal indemnity schedule

In order to solve this problem using optimal control, we introduce the following state variable:[1]

$$\text{(4)} \qquad \Gamma(\hat{x}) = -\int_0^{\hat{x}} \left(I(x) + C\left(\left|I(x)\right|\right)\right) f(x) dx \,.$$

The initial condition is that $\Gamma(0) = 0$. The terminal condition reads as $\Gamma(L) = -P$, which corresponds to a zero-profit constraint for the insurer, i.e. the premiums have to cover the insurer's expected expenditures. The corresponding Hamiltonian is:

$$\text{(5)} \qquad H = U\left(w - P - x + I(x)\right) f(x) - \lambda(x)\left(I(x) + C\left(\left|I(x)\right|\right)\right) f(x) \,.$$

Since the Hamiltonian does not depend on the state variable, it follows from Pontryagin's maximum principle that $\lambda'(x) = -\dfrac{\partial H}{\partial \Gamma} = 0$, i.e. λ is a constant. To find the optimal indemnity schedule, the Hamiltonian is differentiated w.r.t. $I(x)$. After rearranging terms, one has:

$$\text{(6)} \qquad U'\left(w - P - x + I(x)\right) = \lambda\left(1 + C'\left(\left|I(x)\right|\right) \cdot \operatorname{sgn}(I(x))\right).$$

For negative (positive) indemnities, this can be simplified to

$$\text{(7)} \qquad U'\left(w - P - x + I(x)\right) = \lambda\left(1 - C'\left(-I(x)\right)\right) \qquad \text{and}$$

$$\text{(8)} \qquad U'\left(w - P - x + I(x)\right) = \lambda\left(1 + C'\left(I(x)\right)\right),$$

respectively. Eliminating λ and combining (7) and (8) yields

$$\text{(9)} \qquad \left.\frac{U'\left(w - P - x + I(x)\right)}{\left(1 - C'\left(-I(x)\right)\right)}\right|_{I(x)<0} = \left.\frac{U'\left(w - P - x + I(x)\right)}{\left(1 + C'\left(I(x)\right)\right)}\right|_{I(x)>0} .$$

As can be seen from (9), negative indemnities are restricted: The marginal costs they induce must be lower than 1. For indemnities approaching zero, (9) can be approximated by

$$\text{(10)} \qquad \frac{U'\left(w - P - x_-\right)}{\left(1 - C'(0)\right)} = \frac{U'\left(w - P - x_+\right)}{\left(1 + C'(0)\right)} .$$

[1] For mathematical reference see Chiang (1992).

According to (10) $x_- = x_+$ for $C'(0) = 0$. However, for positive marginal costs $(C'(0) > 0)$, the denominator on the rhs of (10) is greater than the denominator on the lhs. To compensate for this difference, $U'(w - P - x_+)$ must be greater than $U'(w - P - x_-)$. Under the assumption of decreasing marginal utility, this can only be the case if $x_- < x_+$. As negative indemnities are restricted, no positive lower bound x_- can be determined if $C'(0) \geq 1$.

From (9) and (10) follows:

- The optimal indemnity schedule is characterized by a lower bound x_- and an upper bound (i.e. a deductible) x_+. For losses lower than x_- indemnities are negative (the insured pays the insurer); for losses exceeding x_+ indemnities are positive. For losses between x_- and x_+ no transfer between insurer and insured takes place.

- The distance between x_- and x_+ depends on marginal costs at $I(x) = 0$ and the insured's risk aversion. The more risk averse insured, the smaller the range of losses $[x_-, x_+]$ they have to bear completely. If $C'(0) = 0$ the values of x_- and x_+ coincide.

For increasing marginal costs full marginal indemnity is not generally optimal. Instead, $\frac{\partial I(x)}{\partial x} \leq 1$. This can be shown by differentiating (6) w.r.t. x:

(11)
$$U''(w - P - x + I(x)) \cdot \left(-1 + \frac{\partial I(x)}{\partial x}\right) = \lambda\left(C''(|I(x)|) \cdot \frac{\partial I(x)}{\partial x}\right).$$

Substituting for λ from (6) and solving for the marginal indemnity $\frac{\partial I(x)}{\partial x}$ yields:

(12)
$$\frac{\partial I(x)}{\partial x} = \frac{(1 + C'(|I(x)|) \cdot \operatorname{sgn}(I(x))) \cdot U''(A)}{(1 + C'(|I(x)|) \cdot \operatorname{sgn}(I(x))) \cdot U''(A) - U'(A) \cdot C''(|I(x)|)},$$

with $A = w - P - x + I(x)$.

Finally, using the definition of absolute risk aversion $Ra(A) = -\frac{U''(A)}{U'(A)}$:

(13) $$\frac{\partial I(x)}{\partial x} = \frac{Ra(A)}{Ra(A) + \frac{C''(|I(x)|)}{1 + C'(|I(x)|) \cdot \operatorname{sgn}(I(x))}}.$$

Remember from (9) that $C'(-I(x)) \leq 1$ for $I(x) < 0$. Therefore, $0 \leq \frac{\partial I(x)}{\partial x} \leq 1$.

Specifically:

- Marginal indemnity increases with insured's risk aversion. As risk aversion approaches infinity, full marginal reimbursement becomes optimal.
- Constant marginal costs are confirmed to be special case of (13) with $C'' = 0$, giving rise to full marginal reimbursement, i.e. $I'(x) = 1$.

However, in general optimal indemnity schedules will call for less than full marginal indemnity and will look like the one shown in figure 1:

Figure 1: Optimal indemnity with increasing marginal costs

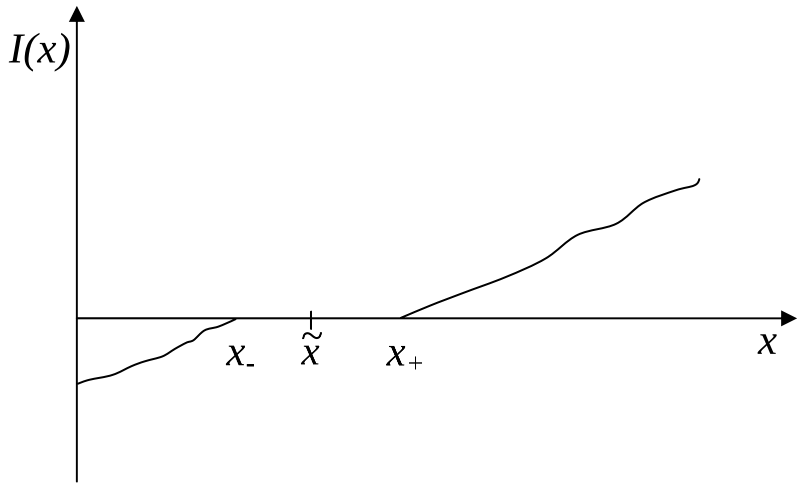

Figure 1 illustrates the results obtained so far if $C'(0) > 0$. From (10) it is known that x_- and x_+ coincide for infinitely risk adverse individuals. In figure 1 this point is labelled $\tilde{x}$. It will be determined in more detail in the next section. However, for less risk averse individuals the two limits x_- and x_+ are on the left hand side and the right hand side of $\tilde{x}$, respectively, if $c'(0) > 0$. For losses $x < x_-$

indemnity is paid from the insured to the insurer (indemnities are negative). For losses $x > x_+$ indemnities are paid from the insurer to the insured (indemnities are positive). Losses between x_- and x_+ are borne by the insured alone. Marginal indemnity is $0 \leq \partial I(x)/\partial x \leq 1$ for all losses: For losses lower than x_-, insured can partly reduce their payments to the insurer, but not for the full amount of the loss; a marginal increase of the loss will only entitle insured to reduce their payments by less than the marginal increase of the loss. For losses greater than x_+ the insured are entitled to receive positive indemnity payments from the insurer. However, marginal indemnity will again be lower than 1 so that insured still have to bear some marginal loss.

3. Is the non-negativity constraint binding?

Having derived the main properties of an insurance contract without the non-negativity constraint on indemnities, it is useful to further explore the terms of the optimal insurance contract. In particular, since $x_- \leq x_+$ the upper bound for x_- and the lower bound for x_+ are of interest. Since x_- and x_+ coincide for infinitely risk adverse insured, both bounds have the same value labelled $\tilde{x}$ in figure 1. However, it may turn out that $\tilde{x}$ is non-positive or very small, in which case the non-negativity constraint would not bind if individuals are not infinitely risk averse. The efficient insurance contracts in this case would be of the deductible type.

To determine $\tilde{x}$ remember that premiums depend on the actuarially fair value of expected (net-)indemnity payments plus administrative cost, which rise as transfers between insurer and insured rise in absolute terms. Consequently, insured's losses in terms of expected wealth are lower if costly transactions between insurer and insured are reduced. While, as noted before, individuals' risk aversion and marginal costs determine the distance between x_- and x_+ as well as the slope of the indemnity function, the critical value $\tilde{x}$ depends on administrative costs alone. Expected transaction costs are minimized for any indemnity schedule by a loss $\tilde{x}$ that minimizes

(14) $$\int_0^L C\left(\left|I\left(x-\tilde{x}\right)\right|\right) f(x)dx \,.$$

Differentiating (14) w.r.t. $\tilde{x}$ yields the necessary condition

(15) $$\int_0^L -C'\left(\left|I\left(x-\tilde{x}\right)\right|\right)\cdot I'\left(x-\tilde{x}\right)\cdot sign(x-\tilde{x})\, f(x)dx = 0 \,,$$

which is easier to read if written as

(16) $$\int_0^{\tilde{x}} C'\left(\left|I\left(x-\tilde{x}\right)\right|\right)\cdot I'\left(x-\tilde{x}\right) f(x)dx = \int_{\tilde{x}}^L C'\left(\left|I\left(x-\tilde{x}\right)\right|\right)\cdot I'\left(x-\tilde{x}\right) f(x)dx \,.$$

Gollier (1987) has shown that if marginal costs are constant, marginal indemnity equals 1 (see also equation (13)) and $\tilde{x}$ always coincides with the median of the loss distribution.[2] Consequently, in this case the non-negativity constraint is never binding if the probability of loss is lower than $1/2$.

However, with non-constant marginal costs, this result no longer holds. Suppose an asymmetric loss distribution with more mass on low losses, as it is characteristic for health insurance. If $\tilde{x}$ coincided with the median, high losses would deviate more from $\tilde{x}$ than low losses, causing higher absolute indemnities than low losses do. Higher indemnities go along with higher marginal costs. To prevent the RHS of (16) to exceed its LHS, one has to chose $\tilde{x}$ such that equation (16) is met again, while the distance between x_- and x_+ is determined by equation (10) and depends on the insured's risk aversion and the marginal costs at $I(x)=0$. Consequently, for sufficiently risk averse insured and low marginal costs at $I(x)=0$, the interval between the two losses x_- and x_+ which is compatible with both equations, (10) and (16) may not include the median but be on its RHS.

Consequently, for strictly convex costs it is not true in general that the non-negativity constraint is not binding if the probability of loss is lower than $1/2$. For

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[2] If the marginal indemnity is 1 and $x_- = x_+ = \tilde{x}$, the term $C'\left(\left|I\left(x-\tilde{x}\right)\right|\right)$ simplifies to $C'\left(\left|x-\tilde{x}\right|\right)$ Assuming non-negative constant marginal costs this allows to rewrite (16) as $C'\left(\left|x-\tilde{x}\right|\right)\cdot\left(\int_0^{\tilde{x}} f(x)dx - \int_{\tilde{x}}^L f(x)dx\right) = C'\left(\left|x-\tilde{x}\right|\right)\cdot\left(2F(\tilde{x})-1\right) = 0$, which is zero if $\tilde{x}$ takes the value of the median.

sharply increasing marginal costs, $\tilde{x}$ may deviate from the median considerably. An imposed non-negativity constraint therefore can be binding more often than Gollier suggests. The effect of a non-negativity constraint to the insured is that they are urged to accept higher marginal costs to reduce the variance of their final wealth. This effect becomes most obvious for insured with risk aversion approaching infinity, inducing full marginal indemnity. In order to stabilize their final income they can only buy full insurance and have to bear the high marginal costs of the high indemnity payments from the insurer. Figure 2 summarizes the range of optimal indemnity schedules for different degrees of risk aversion and different cost functions by highlighting some extreme cases. If marginal costs are zero (cases (a) and (b)) the insured will buy full coverage or buy no insurance at all (if confronted with high fixed costs of the insurance contract, e.g. provision for the agent). If marginal costs are positive but constant (cases (c) and (d)), the insured will always opt for full marginal indemnity. As risk aversion approaches infinity, x_- and x_+ will tend towards the median of the loss distribution as in case (d). Case (e) represents the standard indemnity schedule for non-constant but finite marginal costs and finite risk aversion. Note that depending on the loss distribution $\tilde{x}$ can be on the rhs or the lhs of the median. Consequently, the optimal indemnity schedule for infinitely risk averse individuals cuts the abscissa not at x_{med} (case (f)).

Figure 2: Optimal indemnity schedules without the non-negativity constraint on indemnities

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<td>$0 < Ra(A) < \infty$</td>
<td>$Ra(A) \to \infty$</td>
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<td>$C' = 0$</td>
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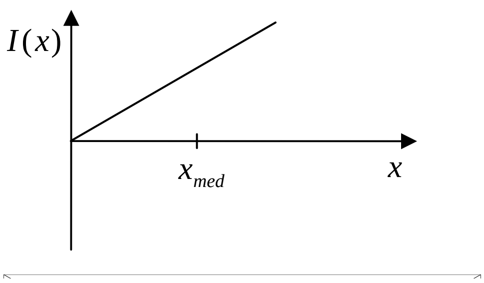
(a)



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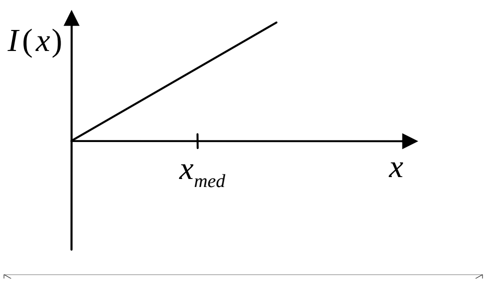

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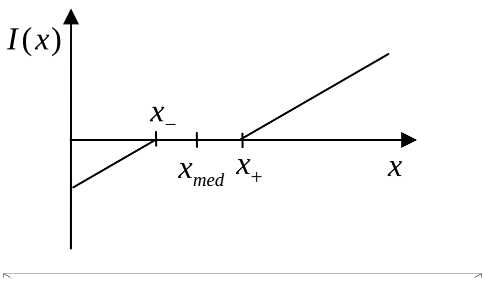
(c)



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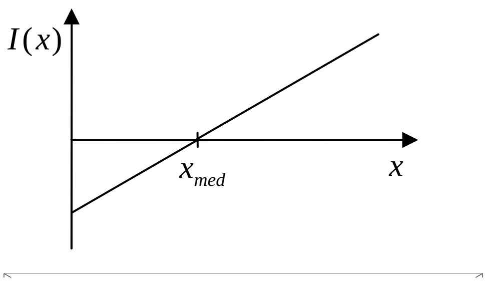

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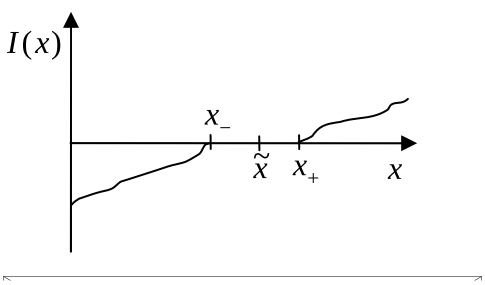
(e)



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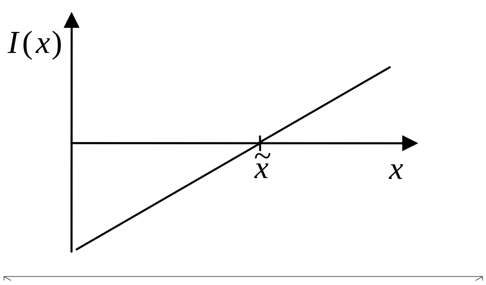
(f)



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4. Self-insurance and self-protection

In the light of the arguments of sections 2 and 3, insurance schedules featuring negative indemnities should be an attractive option for health insurance. However, until now information was assumed to be symmetrically distributed between insurers and insured. This section investigates if the results obtained so far are robust if we assume that insured can affect the loss distribution by hidden actions, which are not observable by the insurers.

Following the terminology introduced by Ehrlich and Becker (1972), individuals can engage in two activities to reduce their expected losses ex-ante: Self-insurance and self-protection. While the former reduces the severity of a loss, self-protection reduces the probability of a loss. If the insurer cannot observe the insured's activities to adjust premiums individually, information becomes asymmetric and moral hazard may arise.

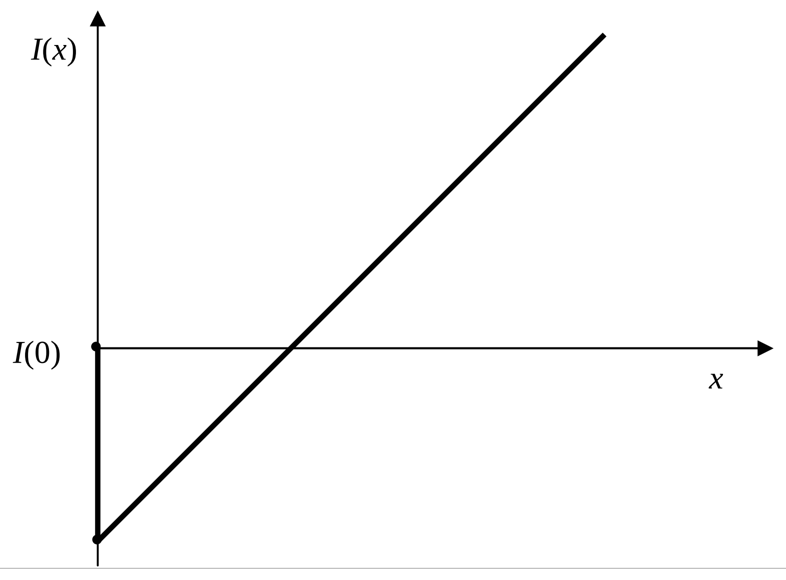
The standard results for insurance contracts imposing the usual non-negativity constraint of indemnities and actuarially fair premiums can be found in the survey by Winter (2000): Given non-decreasing absolute risk-aversion and a distribution of losses that satisfies condition of the monotone likelihood ratio, an indemnity schedule must not feature marginal indemnities exceeding one to encourage the optimal level of self-insurance activities of the insured. Usually, less than full marginal indemnity will turn out to be optimal. To encourage effort to self-protection (or synonymously: prevention), on the other hand, the optimal insurance contract should show full marginal indemnity beyond a non-negative deductible. The intuition behind the latter result is that full insurance cannot be optimal any more because it undermines any effort to reduce the loss probability. Since, on the other hand, the magnitude of a loss does not reveal any additional information about the insured's self-protection activities, the 'penalty' of the insured should be based solely on the occurrence of a loss, rather than on its amount.

For practical applications of negative indemnities to insurance markets that are characterized by asymmetric information it is interesting and relevant to ask how optimal insurance contracts change under moral hazard if the non-negativity constraint on indemnities is dropped. We will discuss both, (1) self-insurance and (2) self-protection, assuming (a) actuarially fair premiums ($C'(|I(x)|)=0$) and (b) constant marginal costs ($C'(|I(x)|)>0,\quad C''(|I(x)|)=0$), which is nothing else than a linear loading.

4.1 Actuarially fair premiums

From sections 2 and 3 it is clear that the optimal insurance schedule under symmetric information and actuarially fair premiums provides full insurance (see also figure 2, cases (a) and (b)). To introduce moral hazard, consider first actuarially fair premiums and self-insurance. Since there are no negative indemnities under symmetric information as long as premiums are fair, the optimal contract under self-insurance changes in exactly the same way as described by Winter, i.e. marginal indemnities should usually be smaller than one, but the optimal contract will show no negative indemnities. In the case of self-protection the optimal contract will not feature a non-negative deductible but will provide full marginal indemnity for all states of loss. However, to provide incentives to self-protection, the indemnity schedule must now prescribe a negative indemnity for low losses and show a discontinuity at the no-loss state: If the insured does not suffer a loss, no transfers to the insurer beside the premium are justified. Figure 3 illustrates.

Figure 3: Actuarially fair premiums and self-protection



However, the indemnity schedule shown in figure 3 gives rise to another incentive problem: Insured who have suffered a small loss are unlikely to report this loss to their insurance, since the only consequence for them would be to receive a bill from their insurers. The solution to this incentive problem is the standard deductible contract.[3] Therefore, if the magnitude of the loss is not observable by the insurer but has to be reported by the insured, the optimal indemnity schedule is not affected by disregarding the non-negativity constraint of indemnities.

4.2 Constant marginal costs

Consider now positive but constant marginal costs (see cases (c) and (d) in figure 2), so that, under symmetric information, the optimal contract would feature full marginal indemnities outside the range $[x_-, x_+]$ and negative indemnities for $x < x_-$. This is not necessarily the case any more if we additionally introduce informational asymmetries. To encourage self-insurance activities the optimal contract may still show positive and negative indemnities but will not provide full marginal indemnity for most amounts of insured loss. Fortunately, as shown before, the same kind of indemnity schedule results if we introduce a non-linear loading and disregard informational asymmetries. Consequently, the basic properties of the insurance contract derived in sections 2 and 3 remain unchanged if we consider self-insurance only.

However, things are more complex if we combine self-protection and premiums containing a loading. To derive the optimal contract, consider a linear loading and a prevention activity e which reduces the probability $p(e)$ of the loss. Let $p'(e) < 0$, $p''(e) > 0$ and $p'(0) = -\infty$. Since effort at prevention is not observable by the insurer, insured will choose their effort e such that

$$
\text{(17)} \qquad \begin{aligned} e^* = \arg\max_e (1-p(e)) \cdot U\big(w - P - e + I(0)\big) \\ + p(e) \cdot \int_{x=0}^{L} U\big(w - P - x - e + I(x)\big) g(x) dx \end{aligned},
$$

which serves as the incentive-compatibility constraint to the new maximization problem. Our assumptions guarantee that condition (17) has an interior solution so that it can be replaced by its first-order condition. Since premiums are assumed to contain a linear loading, the premium function now reads as

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3 See also the model by Townsend (1979).

(18) $$P = p(e)\cdot\left[\int_{x=0}^{L} I(x)f(x)dx + c\int_{x=0}^{L}|I(x)|g(x)dx\right],$$

with c denoting marginal costs. $g(x)$ represents the density function of nonzero losses x. The incentive-compatibility constraint (in the form of its first order condition) and the new premium function can be incorporated into the Hamiltonian. Differentiating the new Hamiltonian w.r.t. the control variable $I(x)$, $x > 0$ yields:

(19) $$\begin{aligned}\frac{\partial H}{\partial I(x)} &= p(e)\cdot U'\big(w-P-x-e+I(x)\big)g(x)\\ &-\lambda\cdot p(e)[1+c\cdot \operatorname{sgn}(I(x))]g(x)\\ &+\mu\cdot\begin{bmatrix} p(e)'U'\big(w-P-x-e+I(x)\big)g(x)\\ -p(e)U''\big(w-P-x-e+I(x)\big)g(x)\end{bmatrix}\end{aligned},$$

with μ as the Lagrange multiplier for first-order condition of the incentive-compatibility constraint.

Applying the same manipulations as in (11) to (13) to (19) proves that the optimal marginal indemnity is still 1 if indemnities are non-zero, as proposed by Gollier in his model for linear costs. However, it turns out that a consistent indemnity schedule featuring positive and negative indemnities might not exist. To show this, (19) is rewritten as

(20) $$\left.\frac{U'(A)\cdot\left(1+\mu\frac{p'(e)}{p(e)}\right)-\mu U''(A)}{1-c}\right|_{I<0} = \lambda = \left.\frac{U'(A)\cdot\left(1+\mu\frac{p'(e)}{p(e)}\right)-\mu U''(A)}{1+c}\right|_{I>0}$$

, with $A = w-P-e-x+I(x)$. For indemnities approaching zero, (20) simplifies to

(21) $$\frac{U'(B)\cdot\left(1+\mu\frac{p'(e)}{p(e)}\right)-\mu U''(B)}{1-c} = \frac{U'(B)\cdot\left(1+\mu\frac{p'(e)}{p(e)}\right)-\mu U''(B)}{1+c},$$

with $B = w-P-e-x$. The lhs of (21) represents the lower bound x_- and the rhs the upper bound x_+. Since marginal indemnity is 1 for all non-zero indemnities, a consistent indemnity schedule can only result if $x_- < x_+$, that is a lower net wealth must result in a higher value of the numerators. Formally: In analogy to (10) the first derivatives of the numerators of (20) w.r.t. B must be negative at an optimum:

(22) $$U''(B)\cdot\left(1+\mu\frac{p'(e)}{p(e)}\right)-\mu U'''(B)<0.$$

Employing the definition of absolute prudence $\left(AP(B)=-\frac{U'''(B)}{U''(B)}\right)$, (22) might be rewritten as $U''(B)+\mu\left(U''(B)\frac{p'(e)}{p(e)}+AP(B)\cdot U''(B)\right)<0$, or

(23) $$\mu\left[\frac{p'(e)}{p(e)}+AP(B)\right]>-1.$$

Let the incentive-compatibility constraint be binding ($\mu>0$). Then, according to (23), a consistent insurance contract featuring negative indemnities for small losses is feasible only if, in equilibrium, the relative effect of prevention on the probability of loss is not too strong (remember that $p'(e)<0$) and/or the insured is sufficiently prudent. In other words: The incentives to further invest in prevention must not be too strong in equilibrium. An individual never has an incentive to invest in prevention if the effect of prevention is not strong enough. As shown by Briys and Schlesinger (1990) and Chiu (2000), the same is true for highly prudent individuals.[4] However, it is somewhat surprising that the shape of the indemnity schedule may change so much that the schedule becomes inconsistent.

If a consistent insurance schedule featuring negative indemnities is not feasible, it is possible to return to an insurance contract with non-negative indemnities only. This contract would, again, show full marginal insurance beyond a non-negative deductible.

The results derived in this section show that negative indemnities are much less attractive if we take into account self-protection. However, this does not preclude that negative indemnities for small losses can be integrated into a useful insurance

4 This is due to the fact that prevention does not reduce the risk of a distribution in terms of first-order or second-order stochastic dominance but increases the downside risk (individuals may suffer a high risk despite their effort at prevention, bearing expenses for prevention in addition to the loss), which individuals dislike the more the more prudent they are.

policy in practice, e.g. for chronically ill persons, if they are already ill at the time they agree to the insurance contract, so that they cannot engage into prevention activities any more. In this case the incentive-compatibility constraint (17) is not binding and the results from sections 2 and 3 apply.

5. Conclusion

The non-negativity constraint on indemnities is common in insurance economics. Keeler (1974) and Gollier (1987) were the first to show that this constraint may be binding under a certain cost function. By deriving the properties of an optimal insurance contract for a broader class of strictly convex cost functions, this paper shows that optimal marginal indemnity will be smaller or equal 1. Furthermore, the optimal contract might contain negative indemnity payments even if probability of loss is less than $1/2$. Both results should make negative indemnities attractive for health insurance, especially for chronically ill individuals.

With respect to moral hazard, on the other hand, the results are somewhat ambiguous: While integrating self-insurance considerations does not change the optimal indemnity schedule substantially, negative indemnities for small losses can turn out to not be optimal any more in the case of pure self-protection. However, this is not to say, that negative indemnities cannot be part of an insurance schedule which is useful in practice, e.g. for chronically ill persons.

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