Learning-by-Doing with Spillovers in Markets with Cournot Competition and Free Entry

by

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Abstract

In this paper we analyze the impact of learning-by-doing with spillovers within a market with free entry where firms engage in Cournot competition. We show that it matters in terms of exit and entry whether firms incur fixed costs only once or in both periods. Further, apart from the well known strategic behavior of firms in Cournot competition, there is an additional strategic effect due to learning. If spillovers are small this distortionary effect mitigates the usual Cournot effect leading to prices closer to the competitive prices. We further analyze the optimal policy for the regulator both for fixed costs incurred only once and fixed costs in both periods. We show that the regulator can, indeed, establish the first best allocation in both cases by offering a menu of subsidies and taxes.

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1 Introduction

Learning-by-doing in imperfect markets has been analyzed thoroughly in previous years. Starting with Spence (1981) almost all authors assumed constant instantaneous costs. Spence characterized the behavior of firms in open-loop and closed-loop equilibria if firms learn privately and firms enter the market at exogenously given entry times. Ghemawat and Spence (1985) showed that if learning spillovers exist two different effects can be observed. First, a negative incentive effect for each firm, because their learning is not entirely private anymore leading to an output reduction. Second, a positive efficiency effect, because as spillovers increase, costs of all firms decrease faster. They show that usually the efficiency effect will dominate, i.e. higher spillovers are better. For Cournot competition with identical firms that can enter whenever they like, Fudenberg and Tirole (1983) were the first to discuss two different equilibrium concepts in a model with private learning-by-doing and constant instantaneous costs. On the one hand a precommitment equilibrium, where all firms decide on their output in the following periods before the first period and a subgame-perfect equilibrium where firms decide strategically and decide on their output levels before each period, on the other hand.

Dasgupta and Stiglitz (1988) question the assumption of identical firms and allow firms to be different. In a duopoly model with private learning and a lower bound for costs (i.e. learning eventually ceases) they show that an initial cost advantage for one firm can lead to a monopoly if the discount rate is large enough. With small discount rates both firms will stay in the market because they anticipate the future oligopoly profits when learning has ceased. For a Betrand-duopoly-model with private learning they show that all output is produced by one firm, but with zero profits. The reason is that the potential competitor forces the producing firms to set the price equal to marginal costs plus the marginal learning benefit in the first period.

Leahy and Neary (1999) in their trade-oligopoly-model with learning draw attention to the point that by investigating the optimal policy of the regulator to distinguish between a precommitment and subgame-perfect equilibrium is not the whole story, but that a third type of equilibrium has to be looked at, where the government cannot precommit to its second period policy either. Cabral and Riordan (1994) and (1997) investigate whether the learning curve induces predatory pricing and show that predatory pricing is not necessarily detrimental to social welfare. Mookherjee and Ray (1991) ask whether private learning influences collusive behavior and show that compared to the no-learning case private learning has no effect on collusion whatsoever. Adding uncertainty, Habermeier (1992) shows that in a duopoly model a lagging firm can catch up and that a firm that will have higher costs after learning has ceased can nevertheless gain monopoly power if it is lucky enough to sell a lot early on. Finally, Jin, Perote-Peña and Troege (2004) assume that firms act boundedly rational and examine the conditions for market exit if firms engage in Bertrand competition with differentiated goods.

Here, we use the same equilibrium concept as in Fudenberg and Tirole (1983), i.e. we assume Cournot competition and subgame-perfect strategies. Additionally, we allow for learning spillovers and assume increasing marginal costs in each period. Except for the market form, the model resembles the perfect competition model in Bläsi and Requate (2005). Important for the results is the insight that it matters whether we assume that firms incur fixed costs only once or in both periods. If fixed costs are paid only once, there will be no late entry or early exit while with fixed costs in both periods both late entry and early exit are possible. We show that apart from the well known strategic behavior of firms in Cournot competition, there is an additional strategic effect due to learning. We show that this effect can be positive if spillovers are small thus dampening the usual Cournot effect yielding prices closer to the case with perfect competition. We further analyze the optimal policy for the regulator both for fixed costs only once and fixed costs in both periods. We show that the regulator can establish the first best allocation in both cases.

We will set up the basic model in Section 2. We then investigate the model with the assumption that fixed costs are incurred once in Section 3, thereby describing the equilibrium concept and the behavior of firms in Subsection 3.1, and characterizing the socially optimal allocations and studying the optimal policy of the regulator in Subsections 3.2 and 3.3. We follow a similar route by investigating the case where firms incur fixed costs in both periods in Section 4. We study the case of late entry in Subsection 4.2 and the case of early exit in Subsection 4.3. We conclude in Section 5.

2 The Basic Model

We consider a closed economy with free market entry where firms engage in Cournot competition. Firms incur learning effects by both their own level of output in the early stage of production (private learning), but also through the level of output by the other firms (learning spillovers). In order to model this we have to consider at least two periods $t = 1, 2$. We denote by n the number of firms and by y_t the output of a typical (symmetric) firm in period t. Total output in period t is written as $Y_t = ny_t$. Further we denote by $C^1(y_1)$ and $C^2(y_2; L)$ the variable production cost in period 1 (without experience) and period 2, respectively, where $L = y_1 + \epsilon(n-1)\tilde{y}_1$ represents the total level of learning, and $(n-1)\tilde{y}_1$ is the output of the remaining $(n-1)$ firms.[2] The extent of learning spillovers is reflected by the parameter ϵ with $0 \leq \epsilon \leq 1$, where $\epsilon = 0$ represents the case of pure private learning and $\epsilon = 1$ the case of complete spillovers. In the latter case, it does not matter for the cost reduction of some firm whether some output is produced by this firm itself or by some other firm.

We assume that the cost functions C^t satisfy the following properties: $C^t_{y_t} > 0$ and $C^t_{y_t y_t} > 0$, i.e. we have positive and increasing marginal costs in each period. Moreover, $C^2_L < 0$ and $C^2_{y_2 L} < 0$, meaning that learning through production (by the own or by the other firms) in period 1 decreases both cost and marginal cost in period 2. Furthermore, $C^2_{LL} > 0$, stating that the marginal effect of learning is decreasing. For technical reasons, in particular second order conditions, we assume overall convexity of C^2, implying $C^2_{LL}C^2_{y_2y_2} - [C^2_{Ly_2}]^2 > 0$ for $y_2, L > 0$. Finally, we assume $C^2(y_2; 0) = C^1(y_1)$, i.e. in the absence of learning the cost functions are identical in both periods. Since we allow for free entry, we assume that there are also fixed costs. We investigate two different cases, first that fixed costs F are incurred only once and second that there are fixed costs F_1 and F_2 in both periods, where we assume for simplicity $F_1 = F_2 = F$. We will study the first case in Section 3, and the second case in Section 4.

A fixed cost incurred only once implies that the firms will either produce or stay out in both periods. The reason is as follows: First, since in the second period fixed costs

[2] We assume $n \geq 1$. Allowing for firm numbers $0 \leq n < 1$ does not make sense here, because learning spillover effects would be negative.

are sunk for firms that have already entered in the first period, new entering firms which have to pay the fixed costs would not be able to compete with the incumbent firms due to higher average costs. Second, since no further firms enter and fixed costs are sunk, we have a standard Cournot market with an exogenous number of firms that are able to accrue a positive oligopoly rent. Therefore it does never pay to leave the market.

Finally, demand for the industry's output is given by an inverse demand function $p_t = P_t(Y_t)$ which is downward sloping and not too convex, i.e. $P_t^{'}(Y_t) < 0$ and

$$P_t^{''} y_t + P_t^{'} < 0 \tag{1}$$

3 Entry is once and for all

3.1 The Equilibrium Concept and the Behavior of Firms

In this section we assume that fixed costs F are incurred only once which implies as pointed out above that entry is once and for all. As Fudenberg and Tirole (1983) have pointed out, one can distinguish between a precommitment equilibrium where firms do not anticipate that the future cost structure of the firms will influence their production decision, on the one hand, and a subgame perfect equilibrium where firms anticapte their influence on both their rivals costs and output and thus adjust their own output accordingly, on the other hand. Since we assume that firms take into account these strategic effects, we do not further analyze open loop equilibria and immediately apply the concept of subgame perfect equilibrium. While firms are not able to precommit we assume that the regulator can indeed make a commitment to levels of tax or subsidy rates to be charged in the second period. In contrast to Leahy and Neary (1999) we therefore do not investigate the case where firms can strategically influence the government's second period decision.

Anticipating the optimal policy to be discussed in Section 3.3 we allow the regulator to pay output subdidies s_1^{out} and s_2^{out} in the first and second period, respectively, and an entry premium s^{ent}. Given this policy the total profit of firm i can be writtten as

$$\pi_i = (P_1(Y_1) + s_1^{out})y_{1,i} - C^1(y_{1,i}) - F + s^{ent} + \delta[(P_2(Y_2) + s_2^{out})y_{2,i} - C^2(y_{2,i}; L_i)] \tag{2}$$

Thus after the regulator has set his policy, the structure of the game among the firms is as follows: In the first period firms decide on whether or not to enter the market and then simultaneously decide on the output to be produced in the first period. By choosing their output they also decide on how much they want to learn. In the second period firms again simultaneously choose quantities.

The game is solved by backward induction. In the second period firms will set marginal costs equal to marginal revenue. The first-order und thus the Nash equilibrium conditions for the second period are given by

$$\pi_{y_{2,i}} \quad = \quad P_2^{'}(Y_2)y_{2,i} + P_2(Y_2) + s_2^{out} - C^2_{y_{2,i}}(y_{2,i}; L_i) = 0 \quad for \quad i = 1, ..., n \tag{3}$$

In the first period firms maximize their profit taking into account (3) and thus taking into consideration their influence on second period output levels. Thus we can set up the maximization problem as a Lagrange maximization problem, where the Lagrange function Λ_i is given by:

$$\Lambda_i = \pi_i + \lambda \pi_{y_{2,i}} + \sum_{j \neq i}^{n} \mu_j \pi_{y_{2,j}} \tag{4}$$

Solving this maximization problem for each firm, exploiting that by symmetry of the firms the equilibrium must be symmetric, and rerranging we obtain the following equilibrium condition (for details see the appendix):

$$P_1 + P_1^{'} y_1 + s_1^{out} = C^1_{y_1} + \delta C^2_L + E, \tag{5}$$

where E is the strategic learning effect defined by

$$E = \frac{\delta C^2_{y_2 L}(n-1)P_2^{'} y_2[(1-2\epsilon)P_2^{'} + \epsilon C^2_{y_2 y_2} + (1-\epsilon)P_2^{''} y_2]}{(C^2_{y_2 y_2} - P_2^{'})(C^2_{y_2 y_2} - (n+1)P_2^{'} - nP_2^{''} y_2)} \tag{6}$$

Thus firms act strategically by setting marginal revenue of the first period equal to net marginal costs plus the strategic effect. The net marginal costs consist of the marginal costs in the first period and the private marginal learning effect, a cost cut induced by first period production on the costs in the second period. The strategic effect is due to the impact the first period output decision of each firm has on marginal costs and output of the competing firms in the second period. Finally, firms will enter the market until profits of all firms are zero:

$$\pi = (P_1(Y_1) + s_1^{out})y_1 - C^1(y_1) - F + s^{ent} + \delta[(P_2(Y_2) + s_2^{out})y_2 - C^2(y_2; L)] = 0 \tag{7}$$

Note that the strategic learning effect is not only due to learning spillovers. Even if learning is only private ($\epsilon = 0$) firms have an incentive to act strategically due to the imperfect competition as was already pointed out by Fudenberg and Tirole (1983).

Since we assume (1), i.e. demand is not too convex, the sign of E depends only on the size of the spillover rate ϵ. Therefore we can state the following result:

Proposition 1 *If condition (1) holds, the strategic effect E is positive (negative, zero) if and only if* $\epsilon > (<, =) \frac{P_2' + P_2'' y_t}{2P_2' + P_2'' y_t - C^2_{y_2 y_2}}$.

It follows that for purely private learning ($\epsilon = 0$) the effect is negative while for complete learning spillovers ($\epsilon = 1$) it is positive. Note that if $E < 0$ the strategic effect resulting from Cournot competition, represented by $P_1' y_1$, is mitigated while with $E > 0$ (i.e. the learning spillovers are sufficiently large) this distortion is exacerbated. Therefore with mainly private learning we will observe prices closer to the competitive price. This confirms the result of Leahy and Neary (1999) who found a negative strategic learning effect with private learning in their trade oligopoly model. Note also that the smaller δ the smaller is the strategic effect with E vanishing if firms are myopic.

3.2 The Social Optimum

Since firms are symmetric and incur increasing marginal costs, an optimal allocation must also be symmetric. Hence we can define welfare W by

$$W = \int_0^{ny_1} P_1(Y)dY - nC^1(y_1) - nF + \delta[\int_0^{ny_2} P_2(Y)dY - nC^2(y_2; L)] \qquad (8)$$

where δ is the social and private discount factor.[3] The social planner maximizes welfare with respect to y_1, y_2 and the optimal number of firms n. The first-order conditions are given by

[3] In general, the social and the private discount factor need not coincide. We abstract from those differences since they are not important for the focus of this paper.

the following equations, where W_{y_1} denotes the partial derivative $\frac{\partial W}{\partial y_1}$ and so on:

$$W_{y_1} = P_1(Y_1) - C^1_{y_1}(y_1) + \delta[-C^2_L(y_2; L)[1 + \epsilon(n-1)]] = 0 \quad (9)$$

$$W_{y_2} = P_2(Y_2) - C^2_{y_2}(y_2; L) = 0 \quad (10)$$

$$\begin{aligned} W_n &= P_1(Y_1)y_1 - C^1(y_1) - F \\ &\quad + \delta[P_2(Y_2)y_2 - C^2(y_2; L) - nC^2_L(y_2; L)\epsilon y_1] = 0 \end{aligned} \quad (11)$$

The first condition states that a typical firm's marginal cost in the first period should be equal to the consumers' marginal willingness to pay for the good plus the social marginal benefit incurred by a cost cut in the second period. The second equation is the usual text book condition stating that in the second period the marginal willingness to pay equals marginal costs. Finally, condition (11) determines (together with conditions (9) and (10)) the optimal number of firms. Interestingly, this condition also contains a learning effect. This is so because the marginal firm which enters the market imposes a positive externality on the other firms by creating a new source of learning.

3.3 The Optimal Policy

By investigating the optimal policy for the regulator we expect three sources of market imperfections. The first one is due to market power in each period, the second one is due to the learning spillovers which distort the output and the entry decision of firms as has already been pointed out in Bläsi and Requate (2005). The third source results from the strategic learning effect E. As is well known, Cournot competition leads to excessive market entry but total output is less than optimal. As we have seen above, the strategic effect E might exacerbate or mitigate this effect. To obtain the first-best allocation we now need the following three instruments, subsidies on output s_1^{out} and s_2^{out} in each period and an entry premium s^{ent}. Equating the conditions (9) - (11) of the social optimum with the equilibrium conditions with free entry (5), (3), and (7), we find that the regulator can establish a first-best allocation if he sets the subsidies in the following way:

$$s_1^{out} = -P_1'(Y_1)y_1 - \delta C_L^2(y_2;L)\epsilon(n-1) + E \lesseqgtr 0 \qquad (12)$$

$$s_2^{out} = -P_2'(Y_2)y_2 > 0 \qquad (13)$$

$$s^{ent} = \underbrace{P_1'(Y_1)y_1^2}_{<0} + \underbrace{\delta P_2'(Y_2)y_2^2}_{<0} - \underbrace{\delta C_L^2(y_2;L)\epsilon y_1}_{>0} - \underbrace{Ey_1}_{\lesseqgtr 0} \lesseqgtr 0 \qquad (14)$$

The first term of the output subsidy given by (12) adjusts for market power in the first period. The second term takes account of the fact that firms do not internalize their positive learning spillovers on other firms. The third term takes account of the distortion arising from the firms strategic behavour. The strategic effect E is positive if ϵ is sufficiently large. Therefore, s_1^{out} will be definitely positive for ϵ not too small. For ϵ small or zero, E can dominate the first two effects. In this case s_1^{out} will be negative, i.e. taxing output in the first period will be optimal. However, this seems a rather unlikely case, as we did not find any examples where E dominates the first two effects.

The output subsidy to be paid in the second period, given by (13), only adjusts for the market power in the second period and is positive. The optimal entry premium or fee s^{ent}, given by (14), can be decomposed into four terms. The first two terms are negative and take account of the usual static excess entry effect of Cournot competition in both periods. The third term is also positive and accounts for too little firms entering the market because of learning spillovers (discussed extensively in Bläsi and Requate (2005)). The forth term takes into consideration the strategic learning effect which may be positive or negative. Thus the sign of the entry premium may be positive, negative or even zero. We can summarize these results in the following proposition:

Proposition 2 *In a market economy with free entry and learning-by-doing with spillovers and firms engaging in Cournot competition the regulator can achieve the first-best allocation by intruducing a per unit subsidy/tax on first period output s_1^{out}, a per unit subsidy on second period output s_2^{out} and an entry premium/fee s^{ent}, where s_1^{out}, s_2^{out} and s^{ent} are determined by (12) - (14).*

Note that apart from those terms in equations (12) and (14) that adjust for the learning spillovers (i.e.$[-\delta C_L^2(y_2;L)\epsilon(n-1)]$ and $[-\delta C_L^2(y_2;L)\epsilon]$, respectively) the regulator's budget is balanced for the other terms. It follows that for only private learning the regulator can reach the social optimum with a balanced budget, since $s_1^{out} y_1 + \delta s_2^{out} y_2 - s^{ent} = 0$ for $\epsilon = 0$.

To illustrate the results we provide an example to compare welfare, output, prices and the number of firms for two different spillover rates between the social optimum, the perfect equilibrium Cournot game, the precommitment Cournot game and perfect competition.

Example 1 *Let* $P_t(Y_t) = 100 - Y_t$, $F = 10$, $\delta = 1$, $C^1(y_1) = 2y_1^2$, $C^2(y_2) = (y_2 - \alpha L)^2 + y_2^2$, *and* $\alpha = 0.02$. *Then for* $\epsilon = 0.1$ *and* $\epsilon = 0.7$ *we obtain the results given in table 1.*

Table 1: A comparison between different market forms

	$\epsilon = 0.7$				$\epsilon = 0.1$			
	SO	SPE	PCE	PC	SO	SPE	PCE	PC
y_1	1.698	1.3768	1.3798	1.692	1.591	1.29538	1.29504	1.589
y_2	1.732	1.4134	1.4160	1.737	1.592	1.30022	1.29992	1.595
n	55.3	67.61	67.48	55.1	59.1	72.234	72.251	59.0
p_1	6.15	6.913	6.895	6.75	5.99	6.429	6.432	6.30
p_2	4.28	4.444	4.456	4.32	5.93	6.080	6.079	5.95
Y_1	93.8	93.087	93.105	93.2	94.0	93.571	93.568	93.7
Y_2	95.72	95.556	95.544	95.69	94.07	93.920	93.921	94.05
E		+0.0399				-0.005		
W	8925.6	8898.05	8898.58	8925.4	8812.92	8788.19	8788.13	8812.87

Notes: SO = Social optimum, SPE = Subgame-perfect equilibrium, PCE = Precommitment equilibrium, PC = Perfect competition, W = Welfare

The corresponding taxes and subsidies that are necessary to achieve the first-best allocations in a Cournot perfect equilibrium game are $s_1^{out} = 2.36$, $s^{ent} = -5.93$, and $s_2^{out} = 1.73$ for $\epsilon = 0.7$, and $s_1^{out} = 1.91$, $s^{ent} = -5.05$, and $s_2^{out} = 1.59$ for $\epsilon = 0.1$. The results especially illustrate that welfare in a Cournot subgame-perfect equilibrium is higher than in a precommitment equilibrium if $E < 0$ and vice versa.

3.4 A note on the spillover rate

In the whole paper we assume that the spillover rate is exogenous and cannot be influenced by the government. Therefore the only thing a government can do to improve welfare is to establish the social optimum with subsidies and/or taxes for this given spillover rate. However, in reality industries might exist where the government could influence the spillover rate, e.g. by forcing firms to open their plants for visits of competitors or by stimulating exchange between managers. If this is possible, a policy to increase the spillover rate could be much more effective than establishing the first-best allocation at a given spillover rate. This is shown in the preceding example 1.[4] Welfare is much higher in all cases with $\epsilon = 0.7$ than with $\epsilon = 0.1$. The reason is intuitive: Even if we have not achieved the socially optimal allocation for the respective spillover rate (say in the subgame-perfect equilibrium with $\epsilon = 0.7$), this allocation can welfare dominate the socially optimal allocation of a low spillover rate (e.g. for $\epsilon = 0.1$), because all firms have lower costs due to the increased spillovers.

4 Fixed Costs in Both Periods - Early Exit and Late Entry

4.1 Extension of the model

We now allow for fixed costs in both periods and assume for simplicity that fixed costs are equal in both periods and therefore are not affected by learning, i.e. $F_1 = F_2 = F$. From Petrakis, Rasmusen and Roy (1997) we know that for purely private learning in a competitive industry with fixed costs and convex cost functions we may observe early exit of firms, i.e. ex ante identical firms behave differently. Some will learn more and stay for two periods in the market whereas others engage in little learning and leave the market early. We refer to the staying firms as S-firms, and to those firms that decide to exit the market after the first period, as X-firms. Bläsi and Requate (2005) have shown that it might be optimal that

[4] In an example Spence (1981) showed the same for different model assumptions. Ghemawat and Spence (1985) show that usually a higher spillover rate increases welfare.

some firms stay while other firms enter late in the second period. To the latter we refer to as E-firms. Those authors have also shown that early exit and late entry cannot occur simultaneously. This is so because due to private learning X-firms have lower costs than E-firms. Obviously it does not make sense that firms with lower cost structures leave the market while at the same time firms with higher costs enter.

For the following analysis we denote by n_s, n_x, and n_e the number of S-firms, X-firms, and E-firms, respectively. Further, we denote by y_1^s and y_2^s the output of the S-firms in period 1 and 2, respectively, by y_x the output of the X-firms in the first period, and by y_e the output of the E-firms in the second period. Assuming symmetry of firms that stay, exit or enter late, we can write $Y_1 = n_s y_1^s + n_x y_x$ to denote total output in the first period and $Y_2 = n_s y_2^s + n_e y_e$ for total output in the second. Note that for cases where some firms enter late, the learning parameter L is different for staying and for entering firms, denoted by $L_s = y_1^s + \epsilon[(n_s - 1)y_1^s]$, and $L_e = \epsilon(n_s y_1^s)$, respectively.

Therefore, the welfare function contains the three different types of firms.

$$
\begin{aligned}
W &= \int_0^{Y_1} P_1(Y)dY - n_s C^1(y_1^s) - n_x C^1(y_x) - (n_s + n_x)F + \\
&\quad \delta[\int_0^{Y_2} P_2(Y)dY - n_s C^2(y_2^s; L_s) - n_e C^2(y_e; L_e) - (n_s + n_e)F]
\end{aligned} \tag{15}
$$

The optimality conditions can now be derived by differentiating welfare with respect to the outputs and the number of firms. As was already pointed out in Bläsi and Requate (2005), it can never be socially optimal to have both X-firms and E-firms in the market. It has been shown, however, that depending on demand and cost functions it can be socially optimal, to have either X-firms and S-firms or E-firms and S-firms, or S-firms only. The first-order-conditions of the welfare optimum are given in the appendix.

In the following we investigate the behavior of the firms in Cournot competition and the optimal policy for the regulator to reach the socially optimal allocation if either early exit or late entry is optimal.

4.2 Late Entry

4.2.1 The Equilibrium Concept and the Behavior of Firms

If late entry is optimal the stock of learning will be different for late entering firms and staying firms as long as $\epsilon \neq 1$. Thus the cost function of the two types of firms and their strategic output reduction will also be different. S-firms have lower costs, produce more and induce a higher distortion than E-firms. Anticipating the optimal policy to be investigated in Section 4.2.2, we allow the regulator to pay output subsidies s_1^{out} and s_2^{out} in the first and second period, respectively, an entry premium or fee s_1^{ent} in the first period and a stepwise fixed fee T to be paid by the firms in the second period. We need the stepwise (two levels) fixed fee T to discriminate implicitly between staying firms and entering firms. Given these instruments the profit of a S-Firm i looks as follows:

$$\begin{aligned} \pi_i^s &= P_1(Y_1)y_{1,i}^s - C^1(y_{1,i}^s) - F + s_1^{out}y_{1,i}^s + s_1^{ent} \\ &\quad + \delta[P_2(Y_2)y_{2,i}^s - C^2(y_{2,i}^s; L_{s,i}) - F + s_2^{out}y_{2,i}^s - T] \end{aligned} \tag{16}$$

The profit of a E-firm k is given by

$$\pi_k^e = P_2(Y_2)y_{e,k} - C^2(y_{e,k}; L_{e,k}) - F + s_2^{out}y_{e,k} - T \tag{17}$$

In an equilibrium with free entry firms' will enter the market such that profits of all firms are zero.

Solving the game backwards again, the first-order conditions for the second period are now represented by

$$\pi_{y_{2,s,i}}^s = P_2^{'}(Y_2)y_{2,i}^s + P_2(Y_2) + s_2^{out} - C_{y_{2,i}^s}^2(y_{2,i}^s; L_{s,i}) = 0 \quad for \quad i = 1,..,n_s \tag{18}$$

$$\pi_{y_{e,k}}^e = P_2^{'}(Y_2)y_{e,k} + P_2(Y_2) + s_2^{out} - C_{y_{e,k}}^2(y_{e,k}; L_{e,k}) = 0 \quad for \quad k = 1,..,n_e \tag{19}$$

Given the number of firms, S-firms maximize their profit in the first period taking into account their influence on second period outputs. As in the preceding section we can set up the maximization problem as a Lagrange maximization problem, where the Lagrange function Λ_i is given by:

$$\Lambda_{i,s} = \pi_{i,s} + \lambda\pi_{y_{2,i}^s} + \sum_{j\neq i}^{n_s} \mu_j \pi_{y_{2,j}} + \sum_{k=1}^{n_e} \gamma_k \pi_{y_{e,k}} \tag{20}$$

The first-order conditions of Λ_i with respect to $y^s_{1,i}$, $y^s_{2,i}$, $y^s_{2,j}$, $y_{e,k}$ λ, μ, and γ are given in the appendix. Solving this problem, following a similar procedure as in Section 3 we obtain one equation which represents the symmetric equilibrium behavior in the first period (for details see appendix):

$$P_1 + P_1^{'} y_1 + s_1^{out} = C^1_{y_1} + \delta C^2_L + E_e \qquad (21)$$

Here E_e is the strategic learning effect. The detailed formula is given by equation (85) in the appendix. Although E_e looks more complicated than E, the only difference is that E_e also takes account of the strategic effects on E-firms. Again, the strategic learning effect is not only due to learning spillovers. Even if learning is only private ($\epsilon = 0$) firms have an incentive to act strategically.

4.2.2 The optimal policy

Let us now characterize the optimal subsidy rates or functions and entry premia if late entry is optimal. We will show that the regulator needs to find a mechanism that induces firms of different types to choose their optimal output levels, respectively. First, we introduce the optimal policy if the regulator can discriminate between firms and restrict their choice of instruments. Then I consider an alternative mechanism where firms choose the optimal output levels voluntarily.

As there are only S-firms in the first period, there is no difference in regulation between the two possibilities. To take into account the Cournot behavior in the first period, the learning spillovers and the strategic learning effect, the regulator should pay an output subsidy s_1^{out} in the first period. With y_1^{s*}, y_e^* etc. we denote the socially optimal levels of output.

$$s_1^{out} \;=\; -P_1^{'}(Y_1^*) y_1^{s*} - \delta\epsilon[C^2_L(y_2^{s*}; L_s^*)(n_s^* - 1) + n_e^* C^2_L(y_e^*, L_e^*)] + E_e \lesseqqgtr 0 \qquad (22)$$

The problem in the second period is that there are two types of firms in the market with different cost structures due to incomplete spillovers. As late entering firms have higher costs than S-firms, we can expect them to produce less output in equilibrium than the staying firms. Because they produce less the strategic output reduction due to Cournot competition in the second period is also less. The marginal strategic incentive is different to the one of the S-firms. The marginal subsidies to be paid to establish the first best allocation are:

$$S_2^{'}(y_2^{s*}) \quad = \quad -P_2^{'}(Y_2^*)y_2^{s*} \tag{23}$$

$$S_2^{'}(y_e^*) \quad = \quad -P_2^{'}(Y_2^*)y_e^* \tag{24}$$

The regulator could as such discriminate between firms because firms enter the market in different periods. Then he could implement the first best allocation by paying linear output subsidies equal to the optimal marginal subsidy rates (23) and (24) and the following entry fees/premia, where $\tilde{s}_1^{ent}$ is to be paid to S-firms in the first period (it might be a fee) and t_2^{ent}, a second period entry fee, is to be paid by E-firms entering in the second period:

$$\tilde{s}_1^{ent} \quad = \quad y_1^{s*}[\underbrace{P_1^{'}(Y_1^*)y_1^{s*}}_{<0} \underbrace{-\delta C^2_{L_s^*}(y_2^{s*};L_s^*)\epsilon}_{>0}] + \underbrace{\delta P_2^{'}(Y_2^*)(y_2^{s*})^2}_{<0} - \underbrace{E_e y_1^{s*}}_{\lesseqgtr 0} \lesseqgtr 0 \tag{25}$$

$$t_2^{ent} \quad = \quad \underbrace{-P_2^{'}(Y_2^*)(y_e^*)^2}_{>0} \tag{26}$$

In the equations (16) - (21) we already anticipated the instruments for a regulation without discrimination. Replacing those instruments by the ones given in equations (22) - (26) and then equating (16), (17), (18), (19), and (21) with (62) - (66) we conclude that a socially optimal allocation is reached.

Paying different output subsidies to different firms may not be legally feasible though. If firms are allowed to pick the output subsidy that suits them then, obviously, all firms would opt for the higher output subsidy, yielding a non-optimal allocation. A way to overcome this problem is to introduce a menu of subsidies and fees, where the firms self-select their subsidies and fees such that their optimal output and entry decision coincide with the socially optimal output levels and number of firms. One possible solution to obtain a subgame perfect Nash-Equilibrium that yields the socially optimal allocation is to offer the following subsidy/fee menu:

1. In the first period, firms receive the uniform output subsidy s_1^{out} as given by (22) and receive an entry premium/fee s_1^{ent} given by

$$s_1^{ent} = y_1^{s*}[\underbrace{P_1^{'}(Y_1^*)y_1^{s*}}_{<0} \underbrace{-\delta C^2_{L_s^*}(y_2^{s*};L_s^*)\epsilon}_{>0}] - \underbrace{E_e y_1^{s*}}_{\lesseqgtr 0} \lesseqgtr 0 \tag{27}$$

2. In the second period, firms receive a uniform output subsidy s_2^{out} and pay a stepwise fixed fee T as given by the following equations:

$$s_2^{out} = -P_2^{'}(Y_2^*)y_2^{s*} \tag{28}$$

$$T = \begin{cases} -P_2^{'}(Y_2^*)y_2^{s*}y_e^* & \text{if } y_2 \leq y_e^* \\ -P_2^{'}(Y_2^*)(y_2^{s*})^2 & \text{if } y_2 > y_e^* \end{cases} \tag{29}$$

Proof: Assume all firms choose the socially optimal levels. We need to show that there is no incentive to deviate for both S-firms and E-firms from that strategy. With this policy regime the first-order conditions of S-firms with respect to y_1^s and y_2^s are equal to zero and profits are zero. Deviating would yield losses. If S-firms reduce their output to y_e^* to save the amount $-P_2^{'}(Y_2^*)y_2^{s*}(y_2^{s*} - y_e^*)$ on the fixed fee T it does not pay off because the received output subsidies decrease by the same amount. For E-firms the zero profit condition holds. However, the first-order condition with respect to y_e is not equal to zero, but positive if y_e^* is chosen. It follows that there is no incentive to produce less. But producing more is also not profitable, because as soon as $y_2 > y_e^*$, E-firms have to pay a higher fixed fee and make losses. Hence, no firm has an incentive to deviate from the socially optimal output levels.

This can be illustrated with the help of the following figure 1. S-firms receive total output

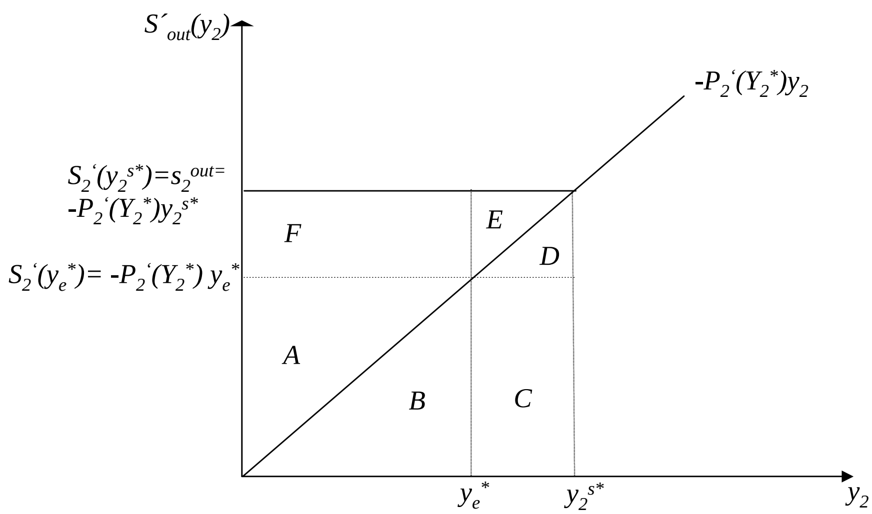

Figure 1: The optimal output subsides and fees in the second period

subsidies of $A + B + C + D + E + F$=$s_2^{out}y_2^{s*}$ and pay a fixed fee of $T = -P_2^{'}(Y_2^*)(y_2^{s*})^2$. E-firms receive total output subsidies of $A+B+F$ and pay a fixed fee of $T = -P_2^{'}(Y_2^*)y_2^{s*}y_e^*$. Producing more than y_e^* would immediately incur a cost of $C+D+E$. This is not profitable.

We can summarize these results in the following proposition:

Proposition 3 *In an economy where firms engage in Cournot competition with free entry, fixed costs in both periods and learning-by-doing with spillovers the regulator can achieve the first-best allocation if late entry is optimal and*

1. *if discrimination between firms is legally feasible by means of a first period output subsidy* s_1^{out}*, linear second period output subsidies according to (23) and (24) and entry fees* $\tilde{s}_1^{ent}$, t_2^{ent}*, where* s_1^{out}, $\tilde{s}_1^{ent}$*, and* t_2^{ent} *are determined by (22), (25), and (26).*
2. *if discrimination is not legally feasible by means of a first period output subsidy* s_1^{out}*, a first period entry fee* s_1^{ent}*, a second period output subsidy* s_2^{out}*, and a second period stepwise fixed fee* T*, where* s_1^{out}, s_1^{ent}, s_2^{out} *and* T *are determined by (22), (27), (28), and (29).*

The following example verifies that late entry is indeed a possible outcome in both an unregulated market where firms engage in Cournot competition and play the subgame-perfect equilibrium strategies and in the social optimum.

Example 2 *Let* $P_t = a - bY_t$, $F = 10$, $\delta = 1$, $C^1(y_1) = 2y_1^2$, $C^2(y_2) = (y_2 - \alpha L)^2 + y_2^2$. *Then we get the results given in table 2.*

Table 2: Late entry in subgame-perfect equilibria

	Late entry, $\epsilon = 0.9$, $a = 10$, $b = 1/9$, $\alpha = 1/15$									
	y_1	y_2	y_e	n_s	n_e	p_1	p_2	Y_1	Y_2	W
SO	2.21	2.22	2.21	4.38	4.00	8.92	7.93	9.70	18.60	24.4
SPE	2.24	2.44	2.43	10.02	0.95	7.51	7.03	22.41	26.73	194.1

Notes: SO = Social optimum, SPE = Subgame-perfect equilibrium, W = Welfare

The difference in welfare values between the social optimum and the subgame-perfect equilibrium in example 2 demonstrates that introducing an optimal policy regime at a given spillover rate can strongly improve welfare.

4.3 Early Exit

4.3.1 The equilibrium concept and the Behavior of Firms

We will now be looking at the case where it is optimal that some firms leave the market after the first period. The learning effect for S-firms now depends on first period output of both staying and early exiting firms, i.e. $L = y_1^s + \epsilon(n_s - 1)y_1^s + \epsilon n_x y_x$. Since in the second period there are only S-firms, the strategic effect is essentially the same as in the case where fixed costs were incurred only once and firms always stayed for both periods. The difference is in the first period, where X-firms are in the market. Assuming symmetry for both types of firms we get the following set of equations which represents the subgame perfect equilibrium with free entry. We again anticipate the optimal policy and allow the regulator to introduce an output subsidy function $S(y_1)$, a constant subsidy s_2^{out} for the second period, a first period entry tax T_{ent}, and finally a second period premium for staying SP. The zero-profit conditions of S-firms and X-firms respectively are:

$$\begin{aligned} \pi^s &= P_1(Y_1)y_{1,s} - C^1(y_{1,s}) - F + S_1(y_1) - T_{ent} \\ &\quad +\delta[P_2(Y_2)y_{2,s} - C^2(y_{2,s}; L) - F + s_2^{out} y_{2,s} + SP] = 0, \end{aligned} \tag{30}$$

$$\pi^x = P_1(Y_1)y_{1,x} - C^1(y_{1,x}) - F + S_1(y_1) - T_{ent} = 0. \tag{31}$$

The first-order condition for X-firms is given by:

$$\pi^x_{y_{1,x}} = P_1^{'} y_{1,x} + P_1 - C^1_{y_{1,x}} + S_1^{'}(y_1) = 0 \tag{32}$$

The first-order conditions of the Lagrange function for the S-firms are - apart from the subsidies - the same as in Section 3.1:

$$P_1^{'} y_{1,s} + P_1 + S_1^{'}(y_1) - C^1_{y_{1,s}} - \delta C^2_L - \lambda C^2_{y_{2,s}L} - (n_s - 1)\mu C^2_{y_{2,s}L}\epsilon = 0 \tag{33}$$

$$\lambda(P_2^{''} y_{2,s} + 2P_2^{'} - C^2_{y_{2,s}y_{2,s}}) + (n-1)\mu(P_2^{''} y_{2,s} + P_2^{'}) = 0 \tag{34}$$

$$\begin{aligned} \delta P_2^{'} y_{2,s} + \lambda(P_2^{''} y_{2,s} + P_2^{'}) + \mu(P_2^{''} y_{2,s} + 2P_2^{'} - C^2_{y_{2,s}y_{2,s}}) & \\ +(n_s - 2)\mu(P_2^{''} y_{2,s} + P_2^{'}) &= 0 \end{aligned} \tag{35}$$

$$\pi_{y_{2,s}} = P_2^{'} y_{2,s} + P_2 - C^2_{y_{2,s}} + s_2^{out} = 0 \tag{36}$$

By solving equations (34) and (35) for λ and μ and plugging the results into (33) we get:

$$P_1 + P_1^{'} y_1^s + S_1^{'}(y_1^s) = C^1_{y_1^s} + \delta C^2_{L_s} + E \tag{37}$$

The strategic effect E is the same as in section 3.1.

4.3.2 Optimal Policy

The optimal policy if early exit is optimal is even more complicated than it is with late entry. From Bläsi and Requate (2005) we know already that with perfect competition the regulator needs a concave output subsidy function if he cannot discriminate between different types of firms. The reason there was that X-firms and S-firms induce different learning spillovers in the second period as the S-firms take already their private learning part into account. Here, one needs to add to this subsidy function the strategic cournot effect and the strategic learning effect of S-firms. For simplicity I take a quadratic subsidy function of the following form, where we first assume that this subsidy function is concave, i.e. $b > 0$ implying $a > 0$.

$$S(y_1) = ay_1 - \frac{b}{2}y_1^2, \tag{38}$$

where

$$a = \frac{Ey_x^* + \delta\epsilon C_L^2(y_2^{s*}, L^*)(n_s^* y_1^{s*} - (n_s^* - 1)y_x^*)}{y_x^* - y_1^{s*}} \tag{39}$$

$$b = P_1^{'}(Y_1^*) + \frac{E + \delta\epsilon C_L^2(y_2^{s*}, L^*)}{y_x^* - y_1^{s*}} \tag{40}$$

The marginal output subsidy function takes the following values at the first best output levels:

$$S_1^{'}(y_1^{s*}) = -\delta\epsilon(n_s^* - 1)C_L^2(y_2^{s*}, L^*) - P_1^{'}(Y_1^*)y_1^{s*} + E \tag{41}$$

$$S_1^{'}(y_x^*) = -\delta\epsilon n_s^* C_L^2(y_2^{s*}, L^*) - P_1^{'}(Y_1^*)y_x^* \tag{42}$$

In the second period, the regulator pays an output subsidy:

$$s_2^{out} = -P_2^{'}(Y_2^*)y_2^{s*} \tag{43}$$

Additionally to the output subsidies, the regulator charges entry taxes:

$$T_{ent} = \underbrace{\frac{1}{2}P_1^{'}(Y_1^*)(y_x^*)^2}_{<0} - \underbrace{\frac{(y_x^*)^2 E}{2(y_x^* - y_1^{s*})}}_{\lesseqgtr 0} + \underbrace{\frac{-\delta\epsilon C_L^2(y_2^{s*}, L^*)(y_x^*)^2}{2(y_x^* - y_1^{s*})}}_{<0} \tag{44}$$

Staying firms receive in the second period (after the first period) a staying premium:

$$SP = \underbrace{-\frac{y_x^* - y_1^{s*}}{2\delta}}_{>0}[\underbrace{P_1^{'}(Y_1^*)}_{<0} - \underbrace{E}_{\lesseqgtr 0}] + \underbrace{\epsilon C_L^2(y_2^{s*}, L^*)(y_x^* - y_1^{s*})}_{>0} + \underbrace{P_2^{'}(Y_2^*)(y_2^{s*})^2}_{<0} \tag{45}$$

Note that this premium can also be a staying tax if the negative effects due to Cournot behavior in both periods dominate E and and the forth term (due to learning spillovers).

Summarizing:

Proposition 4 *In an economy where firms engage in Cournot competition with free entry, fixed costs in both periods and learning-by-doing with spillovers the regulator can achieve the first-best allocation if early exit is optimal and if $b > 0$ by means of a first period output subsidy function $S_1(y_1)$, an output subsidy in the second period s_2^{out}, a first period entry tax T_{ent} and a second period staying premium SP, where $S_1(y_1) = ay_1 - \frac{b}{2}y_1^2$, s_2^{out}, T_{ent} and SP are defined as noted above.*

This policy regime, however, is only an option if $b > 0$ and therefore the subsidy function is concave. If $b < 0$ - $P_1^{'}(Y_1^*)$ could dominate the other effects - the subsidy function is convex and we might run into trouble because firms may be willing to produce an infinite amount of output because subsidies are ever increasing and this possibly faster than costs. Similar to the case of late entry the regulator could establish the socially optimal allocation if he is allowed to discriminate between firms. We need to replace the instruments of proposition 4 with s_{out}^x, $s_{1,out}^s$, s_{ent}^s, t_{out}^x and $s_{2,out}^s$. s_{out}^x, $s_{1,out}^s$, and $s_{2,out}^s$ are output subsidies for the different types in the two periods, s_{ent}^s is an entry premium for S-firms and t_{out}^x is an entry tax for X-firms. By equating the first-order and zero-profit conditions with the conditions of the social optimum (67) - (71) we receive the following optimal subsidies and taxes:

$$s_{out}^x = -P_1^{'}y_{x*} - \delta\epsilon n_s^* C_L^2(y_2^{s*}, L_s^*) \tag{46}$$

$$s_{1,out}^s = -P_1^{'}y_1^{s*} - \delta\epsilon(n_s^* - 1)C_L^2(y_2^{s*}, L_s^*) + E \tag{47}$$

$$s_{ent}^s = P_1^{'}(y_1^{s*})^2 - \delta\epsilon C_L^2 y_1^{s*} - Ey_1^{s*} + \delta P_2^{'}(y_2^{s*})^2 \tag{48}$$

$$t_{out}^x = P_1^{'}(y_{x*})^2 \tag{49}$$

$$s_{2,out}^s = -P_2^{'}y_2^{s*} \tag{50}$$

The following example verifies that early exit is a possible outcome in an unregulated market where firms engage in Cournot competition and play the subgame-perfect equilibrium strategies.

Example 3 *Let* $P_t = a - bY_t$, $F = 10$, $\delta = 1$, $C^1(y_1) = 2y_1^2$, $C^2(y_2) = (y_2 - \alpha L)^2 + y_2^2$. *For* $\epsilon = 0.1$, $a = 300$, $b = 1$ *and* $\alpha = 0.02$ *we get the results given in table 3.*

Table 3: Early exit in subgame-perfect equilibria

	Early exit, $\epsilon = 0.1$, $a = 300$, $b = 1$, $\alpha = 0.02$									
	y_1	y_2	y_x	n_s	n_x	p_1	p_2	Y_1	Y_2	W
SO	2.25	2.28	2.24	128.2	1.52	8.10	7.87	291.9	292.1	85,026.8
SPE	1.84	1.86	1.83	157.1	1.25	9.13	8.06	290.9	292.0	84,917.2

Notes: SO = Social optimum, SPE = Subgame-perfect equilibrium, W = Welfare

It should be noted that different kind of firm types can exist in the social optimum and in an unregulated market. The following example demonstrates that it can be socially optimal that some firms exit early but that in an unregulated market there are no early exiting but late entering firms.

Example 4 *Let* $P_t = a - bY_t$, $F = 10$, $\delta = 1$, $C^1(y_1) = 2y_1^2$, $C^2(y_2) = (y_2 - \alpha L)^2 + y_2^2$. *For* $\epsilon = 0.9$, $a = 16$, $b = 1/9$ *and* $\alpha = 0.05$ *we get the results given in table 4.*

Table 4: Early exit in social optimum, but late entry in subgame perfect equilibrium

	$\epsilon = 0.9$, $a = 16$, $b = 1/9$, $\alpha = 0.05$											
	y_1	y_2	y_x	y_e	n_s	n_x	n_e	p_1	p_2	Y_1	Y_2	W
SO	2.24	3.07	2.24	0	28.4	1.03	0	8.69	6.33	65.7	87.0	644.5
SPE	2.15	2.94	0	2.93	29.5	0	0.08	8.95	6.34	63.5	86.9	643.2

Notes: SO = Social optimum, SPE = Subgame-perfect equilibrium, W = Welfare

5 Conclusion

In this paper we analyzed learning-by-doing with spillovers in a market with free entry where firms engage in Cournot competition. By focusing on subgame-perfect equilibria we showed that there is an additional strategic effect due to learning. This effect was shown to be positive if learning spillovers are small and demand is not too convex. In this case total welfare is higher than in a precommitment Cournot game and vice versa. We analyzed the optimal policy for the regulator and showed that the regulator needs three instruments to get the first-best allocation. Further, we investigated what happens if firms have to pay fixed costs in both periods. Both exit and entry can occur. It follows that there is no natural tendency to concentration in the presence of learning by doing and strategic behavior even if firms are different as was argued by e.g. Dasgupta and Stiglitz (1988). This has also implications for the regulator as in the presence of learning spillovers he cannot simply deduct from increasing or decreasing market concentration whether firms do or do not act strategically. Finally we investigated the optimal policy and showed that although the first best allocation can be obtained it might be too complicated to recommend for implementation. Further research should look on second best subsidy regimes which lead to allocations close to the first best one, but are easier to implement.

6 Appendix

Derivation of equation 5:

The first-order conditions of Λ_i with respect to $y_{1,i}$, $y_{2,i}$, $y_{2,j}$, λ, μ are given by:

$$\Lambda_{y_{1,i}} = P_1' y_{1,i} + P_1 + s_1^{out} - C^1_{y_{1,i}} - \delta C^2_{L_i} - \lambda C^2_{y_{2,i}L} - \sum_{j \neq i}^{n} \mu_j C^2_{y_{2,j}L_i} \epsilon \stackrel{!}{=} 0 \quad (51)$$

$$\Lambda_{y_{2,i}} = \lambda(P_2'' y_{2,i} + 2P_2' - C^2_{y_{2,i}y_{2,i}}) + \sum_{j \neq i}^{n} \mu_j (P_2'' y_{2,j} + P_2') \stackrel{!}{=} 0 \quad (52)$$

$$\begin{aligned} \Lambda_{y_{2,j}} &= \delta P_2' y_{2,i} + \lambda(P_2'' y_{2,i} + P_2') + \mu_j (P_2'' y_{2,j} + 2P_2' - C^2_{y_{2,j}y_{2,j}}) \\ &+ \sum_{k \neq i,j}^{n} \mu_k (P_2'' y_{2,k} + 2P_2') \stackrel{!}{=} 0 \quad for \quad j \neq i \end{aligned} \quad (53)$$

$$\Lambda_\lambda = \pi_{y_{2,i}} \stackrel{!}{=} 0 \quad (54)$$

$$\Lambda_{\mu_j} = \pi_{y_{2,j}} \stackrel{!}{=} 0 \quad for \quad j \neq i \quad (55)$$

Note that (53) and (55) represent $(n-1)$ equations for all $j \neq i$, and since we have n firms, (51)-(55) hold n times. By symmetry, however, all firms behave identically in equilibrium and, therefore, the equilibrium conditions simplify to

$$P_1' y_1 + P_1 + s_1^{out} - C^1_{y_1} - \delta C^2_L - \lambda C^2_{y_2 L} - (n-1)\mu C^2_{y_2 L}\epsilon = 0 \quad (56)$$

$$\lambda(P_2'' y_2 + 2P_2' - C^2_{y_2 y_2}) + (n-1)\mu(P_2'' y_2 + P_2') = 0 \quad (57)$$

$$\begin{aligned} \delta P_2' y_2 + \lambda(P_2'' y_2 + P_2') + \mu(P_2'' y_2 + 2P_2' - C^2_{y_2 y_2}) & \\ +(n-2)\mu(P_2'' y_2 + P_2') &= 0 \end{aligned} \quad (58)$$

$$\pi_{y_2} = 0 \quad (59)$$

Finally, as firms enter the market such that total profits are zero:

$$\pi = P_1(Y_1)y_1 - C^1(y_1) - F + \delta[P_2(Y_2)y_2 - C^2(y_2; L)] = 0 \quad (60)$$

Solving equations (57) and (58) for λ and μ and plugging the results into equation (56) we get equation (5)

$$P_1 + P_1' y_1 + s_1^{out} = C^1_{y_1} + \delta C^2_L + E, \quad (61)$$

First-order conditions of the welfare function in Subsection 4.1:

If there are only E-firms and S-firms, the first-order conditions are:

$$\begin{aligned}
W_{y_1^s} &= P_1(Y_1) - C^1_{y_1^s}(y_1^s) + \delta[-C^2_L(y_2^s; L_s)(1+\epsilon(n_s-1)) - n_e C^2_{L_e}(y_e; L_e)\epsilon] = 0 && (62)\\
W_{y_e} &= P_2(Y_2) - C^2_{y_e}(y_e, L_e) = 0 && (63)\\
W_{y_2^s} &= P_2(Y_2) - C^2_{y_2^s}(y_2^s; L_s) = 0 && (64)\\
W_{n_s} &= P_1(Y_1)y_1^s - C^1(y_1^s) - F && (65)\\
&\quad +\delta[P_2(Y_2)y_2^s - C^2(y_2^s; L_s) - n_s C^2_{L_s}(y_2^s; L_s)\epsilon y_1^s - n_e C^2_{L_e}(y_e; L_e)\epsilon y_e - F] = 0\\
W_{n_e} &= P_2(Y_2)y_e - C^2(y_e, L_e) - F = 0 && (66)
\end{aligned}$$

If there are only X-firms and S-firms, the first-order conditions are:

$$\begin{aligned}
W_{y_1^s} &= P_1(Y_1) - C^1_{y_1^s}(y_1^s) + \delta[-C^2_L(y_2^s; L)[1+\epsilon(n_s-1)]] = 0 && (67)\\
W_{y_x} &= P_1(Y_1) - C^1_{y_x}(y_x) + \delta[-C^2_L(y_2^s; L)\epsilon n_s] = 0 && (68)\\
W_{y_2^s} &= P_2(Y_2) - C^2_{y_2^s}(y_2^s; L) = 0 && (69)\\
W_{n_s} &= P_1(Y_1)y_1^s - C^1(y_1^s) - F && (70)\\
&\quad +\delta[P_2(Y_2)y_2^s - C^2(y_2^s; L) - n_s C^2_L(y_2^s; L)\epsilon y_1^s - F] = 0\\
W_{n_x} &= P_1(Y_1)y_x - C^1(y_x) - F + \delta[-n_s C^2_L(y_2^s; L)\epsilon y_x] = 0 && (71)
\end{aligned}$$

The first-order conditions of Λ_i in the case of late entry with respect to $y_{1,i}^s$, $y_{2,i}^s$, $y_{2,j}^s$, $y_{e,k}$ λ, μ, and γ:

$$
\begin{aligned}
\Lambda_{y^s_{1,i}} &= P_1^{'} y^s_{1,i} + P_1 - C^1_{y^s_{1,i}} - \delta C^2_{L_{i,s}} - \lambda C^2_{y^s_{2,i} L_s} + s_1^{out} &(72)\\
&\quad - \sum_{j \neq i}^{n_s} \mu_j C^2_{y^s_{2,j} L_{j,s}} \epsilon - \sum_{k}^{n_e} \gamma_k C^2_{y_{e,k} L_{k,e}} \epsilon \stackrel{!}{=} 0 \\
\Lambda_{y^s_{2,i}} &= \lambda(P_2^{''} y^s_{2,i} + 2P_2^{'} - C^2_{y^s_{2,i} y^s_{2,i}} &(73)\\
&\quad + \sum_{j \neq i}^{n_s} \mu_j (P_2^{''} y^s_{2,j} + P_2^{'}) + \sum_{k}^{n_e} \gamma_k (P_2^{''} y_{e,k} + P_2^{'}) \stackrel{!}{=} 0 \\
\Lambda_{y^s_{2,j}} &= \delta P_2^{'} y^s_{2,i} + \lambda(P_2^{''} y^s_{2,i} + P_2^{'}) + \mu_j (P_2^{''} y^s_{2,j} + 2P_2^{'} - C^2_{y^s_{2,j} y^s_{2,j}}) &(74)\\
&\quad + \sum_{h \neq i,j}^{n_s} \mu_h (P_2^{''} y^s_{2,h} + 2P_2^{'}) + \sum_{k}^{n_e} \gamma_k (P_2^{''} y_{e,k} + 2P_2^{'}) \stackrel{!}{=} 0 \\
\Lambda_{y_{e,k}} &= \delta P_2^{'} y_{e,k} + \lambda(P_2^{''} y_{e,k} + P_2^{'}) + \gamma_k (P_2^{''} y_{e,k} + 2P_2^{'} - C^2_{y_{e,k} y_{e,k}}) &(75)\\
&\quad + \sum_{i}^{n_s} \mu_i (P_2^{''} y^s_{2,i} + 2P_2^{'}) + \sum_{i \neq k}^{n_e} \gamma_i (P_2^{''} y_{e,i} + 2P_2^{'}) \stackrel{!}{=} 0 \\
\Lambda_\lambda &= \pi_{y^s_{2,i}} \stackrel{!}{=} 0 &(76)\\
\Lambda_{\mu_j} &= \pi_{y^s_{2,j}} \stackrel{!}{=} 0 &(77)\\
\Lambda_{\gamma_k} &= \pi_{y_{e,k}} \stackrel{!}{=} 0 &(78)
\end{aligned}
$$

Again, there are actually $(n_s - 1)$ equations for all $j \neq i$ in the form of equations (74) and (77) and n_e equations in the form of equations (75) and (78). In equilibrium all firms of the respective type behave identically and therefore the first-order conditions of Λ_i simplify to

$$
\begin{aligned}
P_1^{'} y^s_1 + P_1 - C^1_{y^s_1} - \delta C^2_{L_s} + s_1^{out} - \lambda C^2_{y^s_2 L_s} & &(79)\\
-(n_s - 1)\mu C^2_{y^s_2 L_s} \epsilon - n_e \gamma C^2_{y_e L_e} \epsilon &= 0 \\
\lambda(P_2^{''} y^s_s + 2P_2^{'} - C^2_{y^s_2 y^s_2}) + (n_s - 1)\mu(P_2^{''} y^s_2 + P_2^{'}) & &(80)\\
+ n_e \gamma (P_2^{''} y_e + P_2^{'}) &= 0 \\
\delta P_2^{'} y^s_2 + \lambda(P_2^{''} y^s_2 + P_2^{'}) + \mu(P_2^{''} y^s_2 + 2P_2^{'} - C^2_{y^s_2 y^s_2}) & &(81)\\
+(n_s - 2)\mu(P_2^{''} y^s_2 + P_2^{'}) + n_e \gamma (P_2^{''} y_e + P_2^{'}) &= 0 \\
\delta P_2^{'} y^s_2 + \lambda(P_2^{''} y^s_2 + P_2^{'}) + \gamma(P_2^{''} y_e + 2P_2^{'} - C^2_{y_e y_e}) & &(82)\\
+(n_s - 1)\mu(P_2^{''} y^s_2 + P_2^{'}) + (n_e - 1)\gamma(P_2^{''} y_e + P_2^{'}) &= 0 \\
\pi^s_{y^s_2} &= 0 &(83)\\
\pi^e_{y_e} &= 0 &(84)
\end{aligned}
$$

Solving equations (80) - (82) for λ, μ and γ and plugging the results into equation (79) we obtain equation (21). The strategic effect E^e is given by

$$E^e = \frac{N}{D}, \tag{85}$$

where the numerator N and the denominator D are given as follows, where we write for short $P_1^{'} = P_1^{'}(Y_1^*)$ and so on:

$$\begin{aligned} N &= \delta P_2^{'} y_2^s (C^2_{y_e L_e} \epsilon n_e (C^2_{y_2^s y_2^s} - P_2^{'})(C^2_{y_2^s y_2^s} - 2P_2^{'} - P_2^{''} y_2^s) \\ &+ C^2_{y_e y_e} C^2_{y_2^s L_s} (n_s - 1)(P_2^{'} + P_2^{''} y_2^s + \epsilon (C^2_{y_2^s y_2^s} - 2P_2^{'} - P_2^{''} y_2^s)) \\ &+ C^2_{y_2^s L_s} [P_2^{'}(-P_2^{'}(n_s + n_e - 1) \\ &+ (n_s - 1)(\epsilon(2P_2^{'} + P_2^{''} y_2 + C^2_{y_2 y_2} P_2^{'}) - P_2^{''} y_2) \\ &- n_e P_2^{''} y_e - C^2_{y_2 y_2}(P_2^{'} + P_2^{''} y_e)] \end{aligned} \tag{86}$$

$$\begin{aligned} D &= (C^2_{y_2^s y_2^s} - P_2^{'})[C^2_{y_e y_e}(C^2_{y_2^s y_2^s} - (n_s + 1)P_2^{'} - n_s P_2^{''} y_2^s) \\ &- C^2_{y_2^s y_2^s}((n_e + 1)P_2^{'} + n_e P_2^{''} y_e) \\ &+ P_2^{'}((n_e + 1)P_2^{'} + n_s P_2^{'} + P_2^{''}(n_s y_2^s + n_e y_e))] \end{aligned} \tag{87}$$

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